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In the name of Allah

Effects of Dark Energy on the structure formation

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Outline:

- Review on the Cosmology
- Dark energy evidences
- Structure Formation
  - Perturbation theory
  - Evolution of perturbations
- ISW effect
  - Effect of sound speed

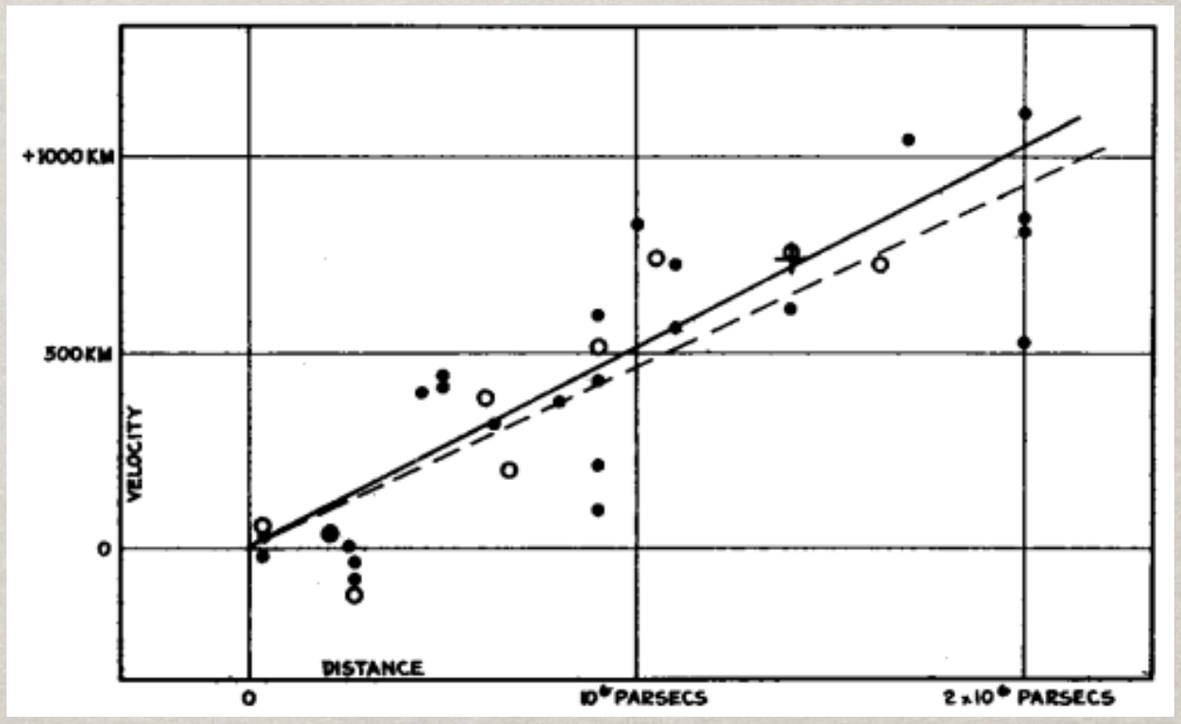
Cosmology

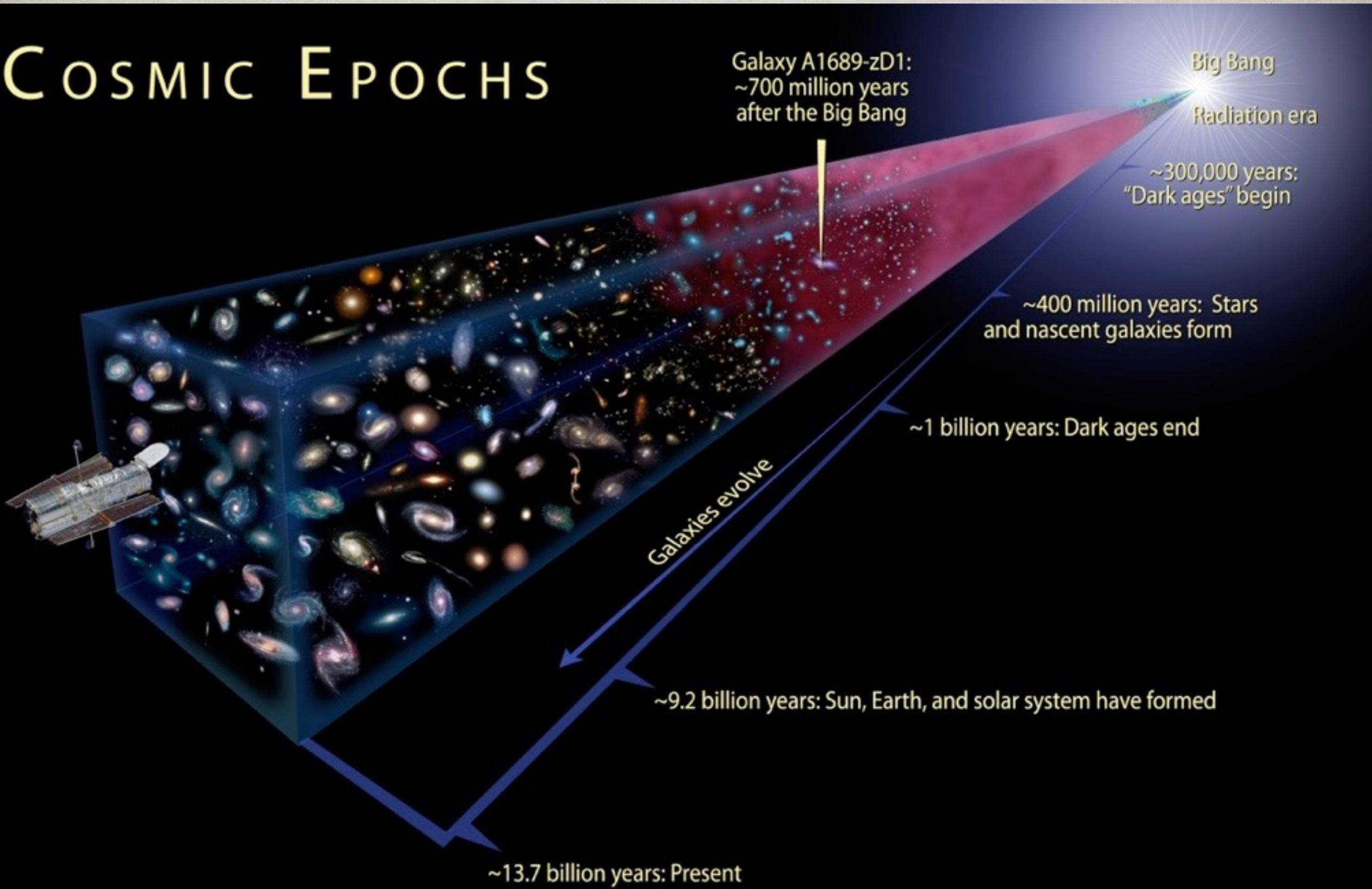
Hubble expansion

$$z \equiv \frac{\Delta\lambda}{\lambda_{em}} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

$$v = H_0 d$$

$$z = \frac{H_0}{c} d$$




Cosmic Epochs
Galaxy A1689-zD1:
~700 million years
after the Big Bang
Big Bang
Radiation era
~300,000 years:
"Dark ages" begin
~400 million years: Stars
and nascent galaxies form
~1 billion years: Dark ages end
Galaxies evolve
~9.2 billion years: Sun, Earth, and solar system have formed
~13.7 billion years: Present

Standard Cosmology

$$\text{Lagrangian:} \quad \mathcal{L} = \frac{1}{16\pi G}\mathcal{R}\sqrt{-g}$$

$$\text{Einstein Eq:} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\text{Metric:} \quad ds^2 = c^2dt^2 - a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

$$ds^2 = c^2dt^2 - a^2(t)\left(d\chi^2 + Sinn^2(\chi)\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

$$Sinn(\chi) \equiv \begin{Bmatrix} \sinh(\chi) & \text{for } K=-1 \\ \chi & \text{for } K=0 \\ \sin(\chi) & \text{for } K=+1 \end{Bmatrix}$$

Cosmological Dynamics

Energy-Momentum Tensor : $T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$

First Fridmann Eq : $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$

Second Fridmann Eq : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}$

Hubble Parameter: $H = \frac{\dot{a}}{a}$

Continuity Eq : $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$

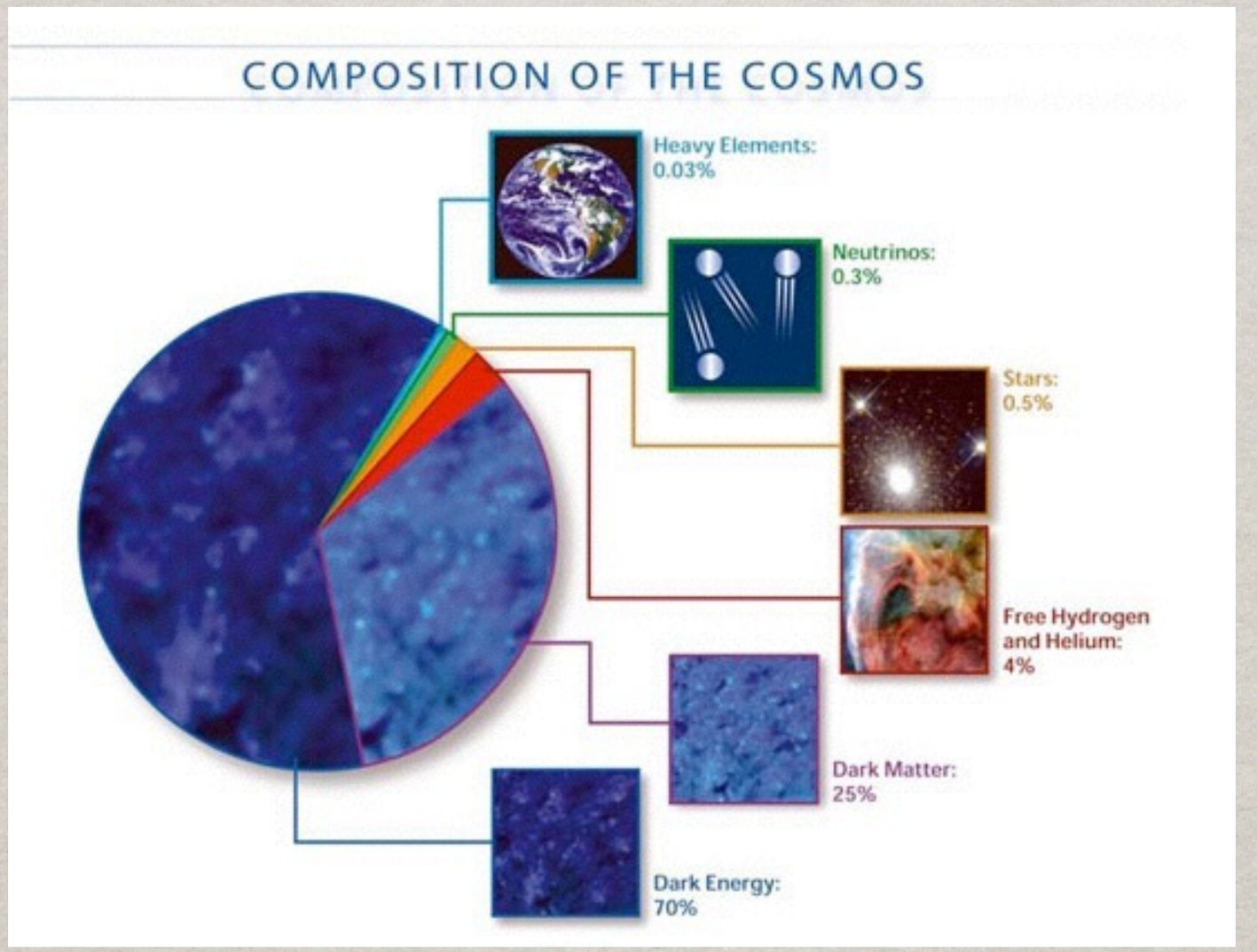
Equation of state: $P = w\rho$

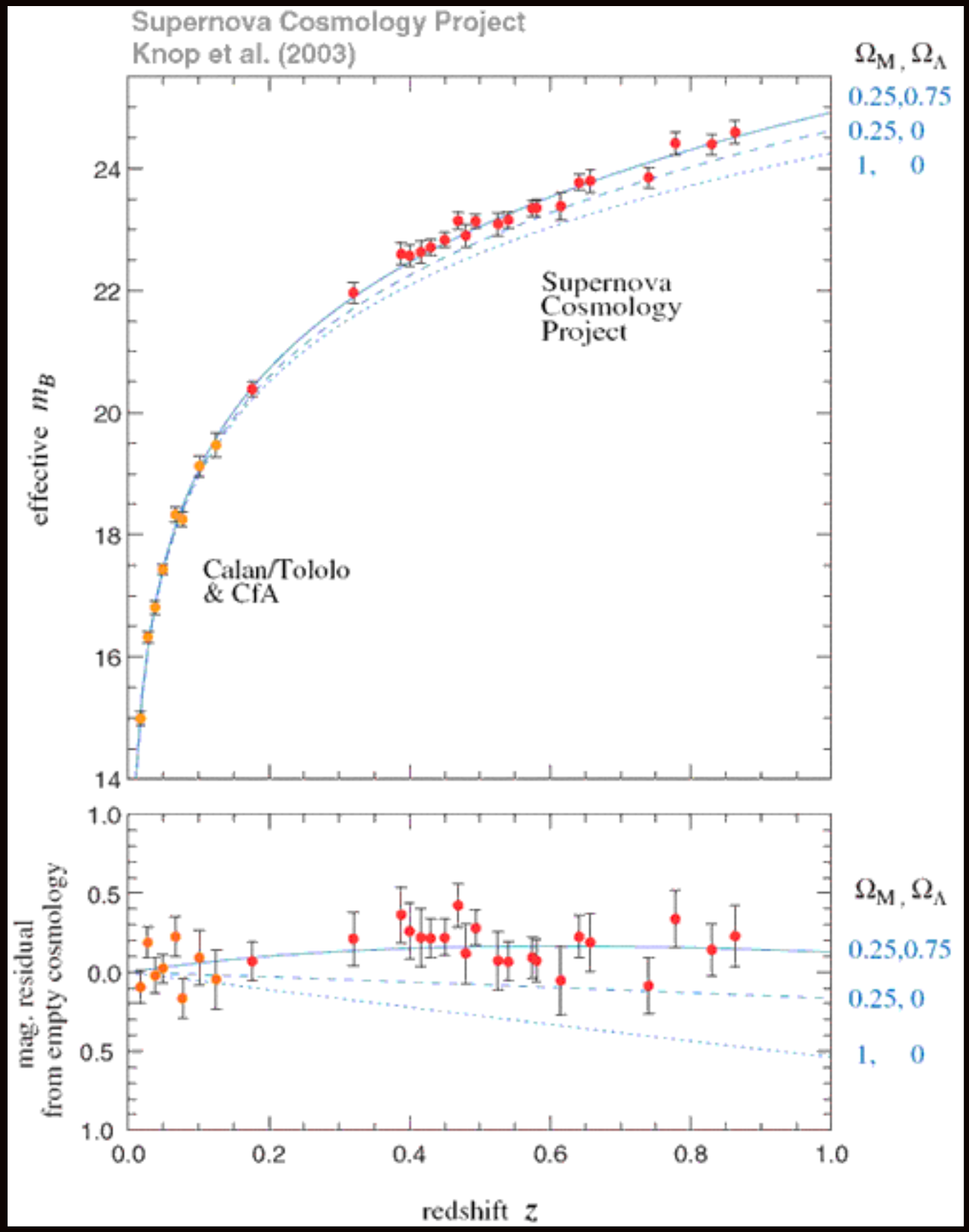
Radiation: $w = \frac{1}{3}$

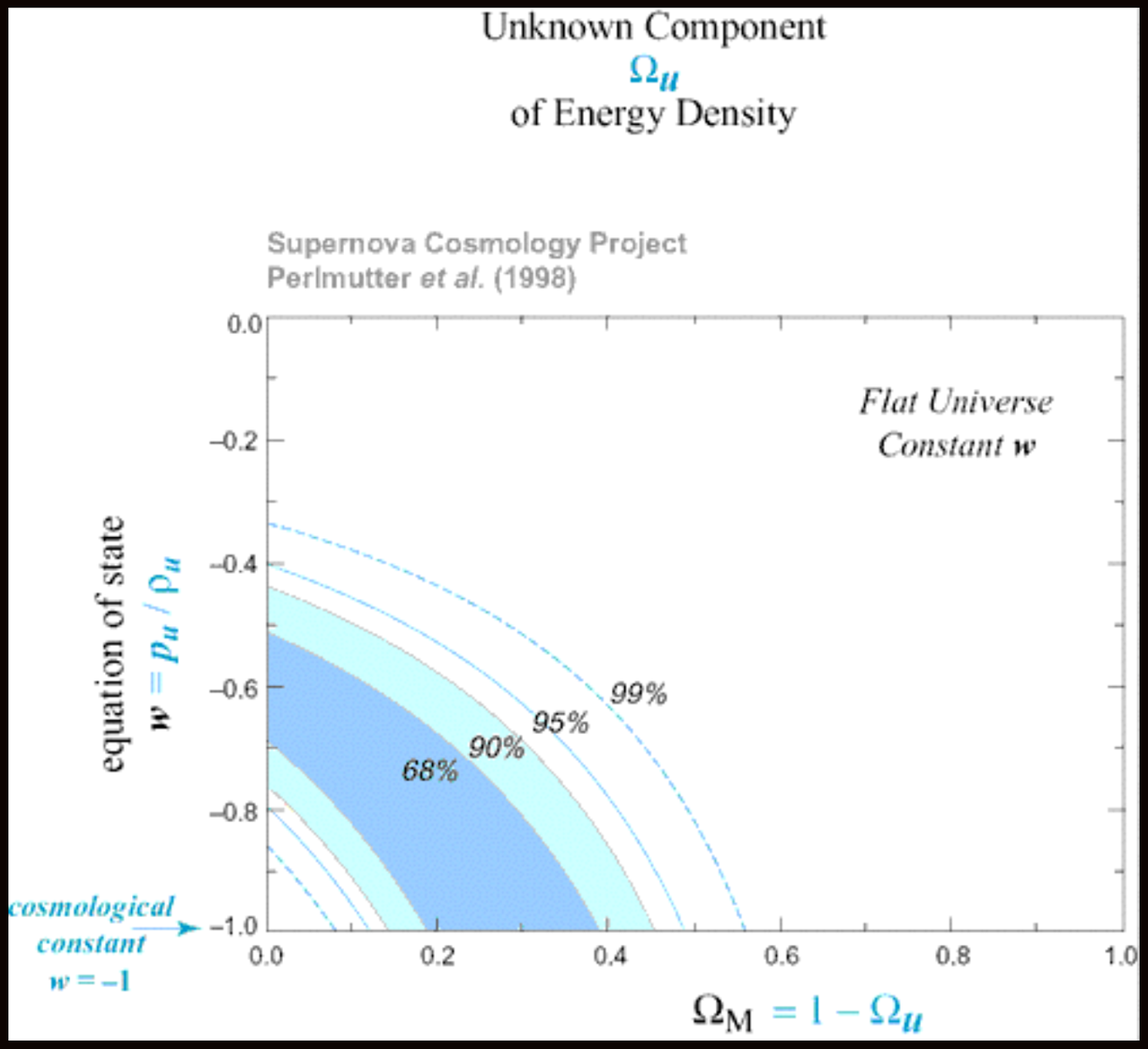
Dark Matter: $w = 0$

Dark Energy: $w = -1$

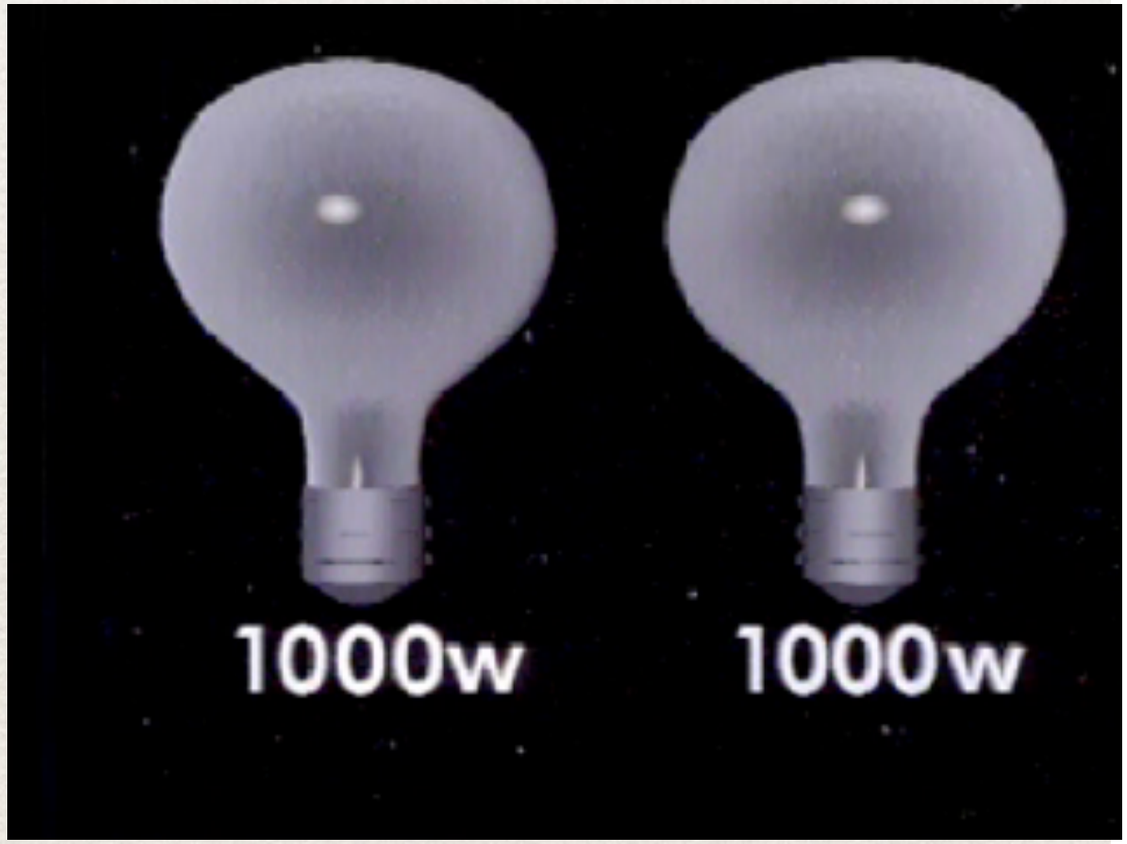
Composition of the universe

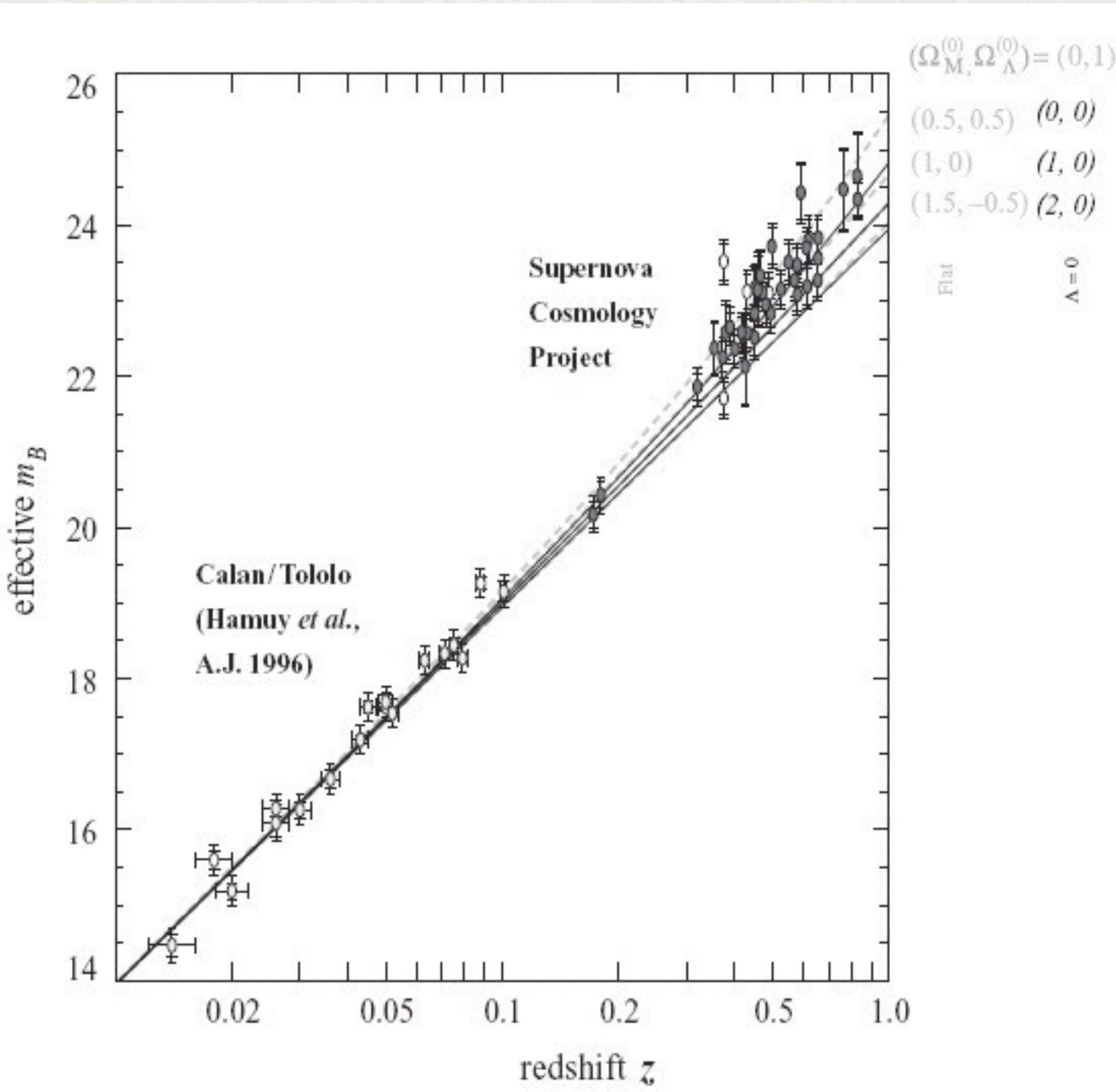


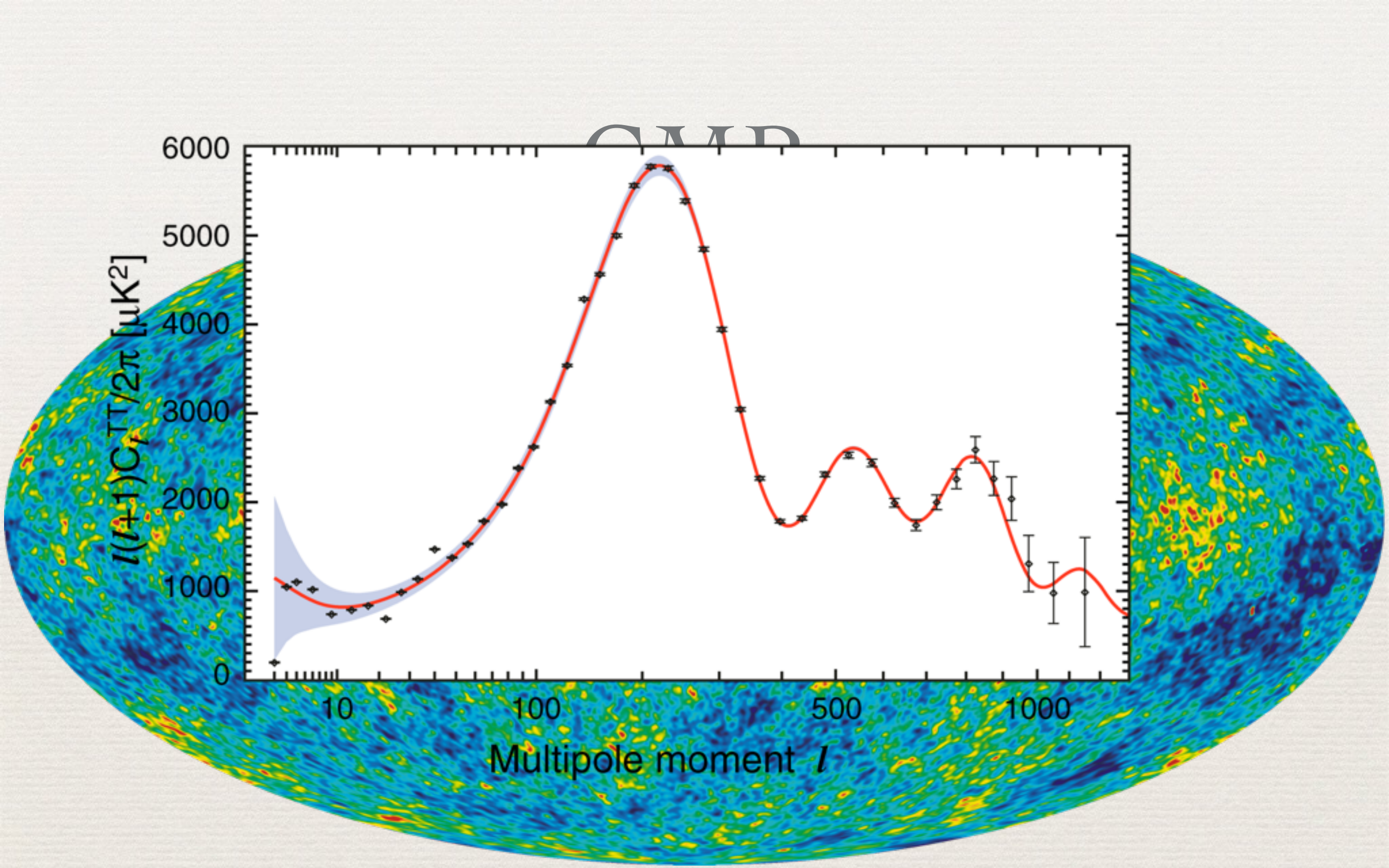

Supernova Cosmology Project
Knop et al. (2003)
Ω_M , Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
24
22
20
18
16
14
mag. residual from empty cosmology
1.0
0.5
0.0
0.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0
redshift z


Unknown Component
Ω_u
of Energy Density
Supernova Cosmology Project
Perlmutter *et al.* (1998)
Flat Universe
Constant w
equation of state
$w = p_u / \rho_u$
0.0
–0.2
–0.4
–0.6
–0.8
–1.0
99%
95%
90%
68%
cosmological constant
$w = -1$
0.0
0.2
0.4
0.6
0.8
1.0
$\Omega_M = 1 - \Omega_u$

DE evidences: SN Ia



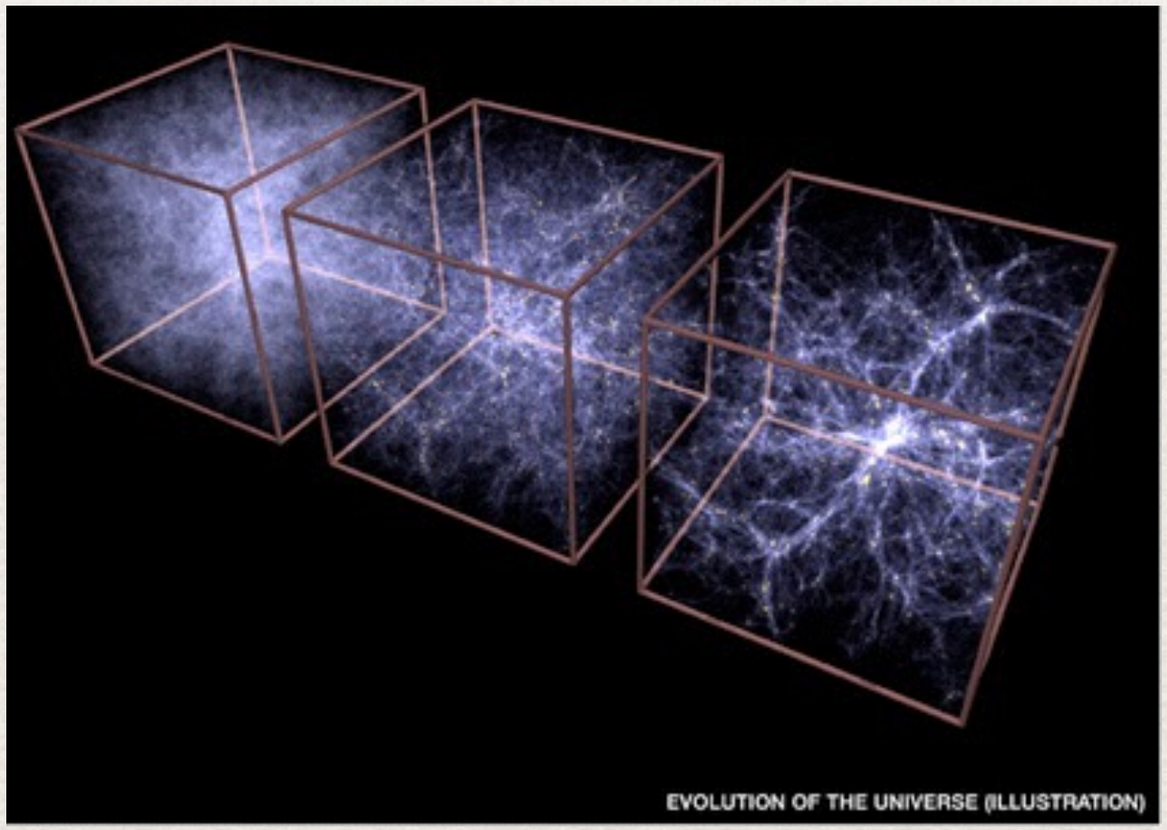



6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l^{TT}/2\pi$ [μK^2]
10
100
500
1000
Multipole moment l

Dark Energy

- Difficult to understand with standard physics
- Different models, different equation of state: $w \equiv p / \rho$
- Different sound speed: $c_s^2 \equiv \frac{dP}{d\rho} = \frac{\dot{P}}{\dot{\rho}}$

Structure Formation



Filaments

Perturbing General Relativity

$$ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = a^2(-d\eta^2 + \delta_{ij} dx^i dx^j) \longrightarrow \text{Background}$$

$$\mathcal{H} \equiv \frac{1}{a}\frac{da}{d\eta} = Ha$$

Perturbed metric in Newtonian gauge $\quad g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij} dx^i dx^j]$$

$$G^\mu_\nu = G^{\mu(0)}_\nu + \delta G^\mu_\nu$$
$$T^\mu_\nu = T^{\mu(0)}_\nu + \delta T^\mu_\nu$$
$$G^{\mu(0)}_\nu = 8\pi G T^{\mu(0)}_\nu$$
$$\delta G^\mu_\nu = 8\pi G \delta T^\mu_\nu$$

$$\rho(t,x) = \overline{\rho}(t) + \delta\rho(t,x)$$

Single fluid

$$\delta \equiv \frac{\delta\rho}{\rho}, \qquad \theta \equiv \nabla_i v^i \qquad \rho(t,x) = \overline{\rho}(t) + \delta\rho(t,x)$$

$$w \equiv p / \rho$$

$$c_s^2 \equiv \frac{dP}{d\rho} = \frac{\dot{P}}{\dot{\rho}} \qquad c_s^2 = 1 \quad \rightarrow \text{Quintessence} \qquad \lambda_J = c_s\sqrt{\frac{\pi}{G\rho}}$$

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}\delta = 0$$

$$f \equiv \frac{d\ln\delta_m}{d\ln a} = \Omega_m^{\gamma} \quad \text{for} \;\; \Lambda CDM \rightarrow \gamma \approx 0.55$$

Evolution of Perturbations

$$k^2\Phi + 3\mathcal{H}(\Phi' - \mathcal{H}\Psi) = 4\pi G a^2 \rho\delta$$

$$k^2(\Phi' - \mathcal{H}\Psi) = -4\pi G a^2 (1+w)\rho\theta$$

$$\Psi = -\Phi$$

$$\Phi'' + 2\mathcal{H}\Phi' - \mathcal{H}\Psi' - (\mathcal{H}^2 + 2\mathcal{H}')\Psi = -4\pi G a^2 c_s^2 \rho\delta$$

$$\delta' + 3\mathcal{H}(c_s^2 - w)\delta = -(1+w)(\theta + 3\Phi')$$

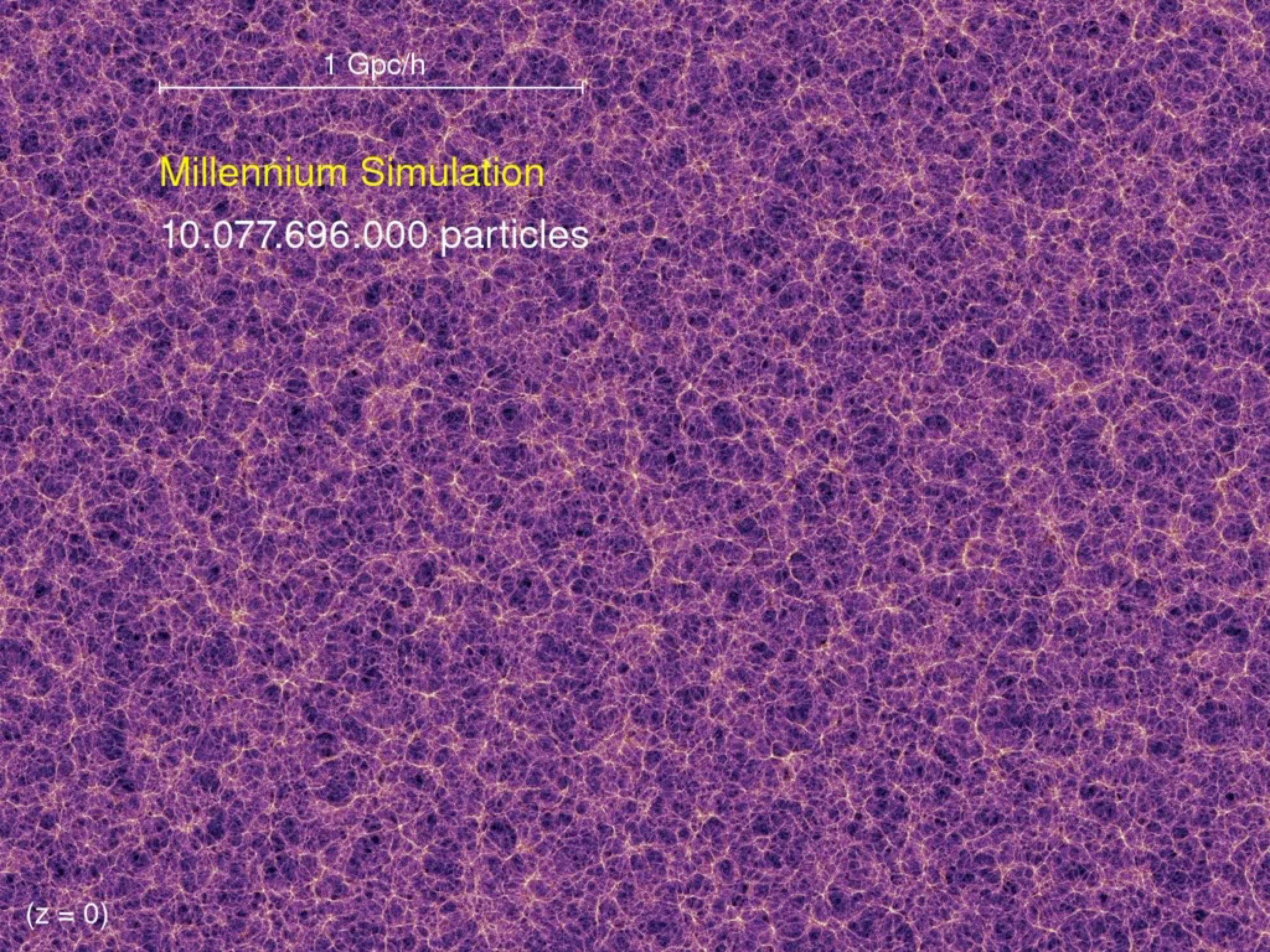
$$\theta' + \left[\mathcal{H}(1-3w) + \frac{w'}{1+w}\right]\theta = k^2\left(\frac{c_s^2}{1+w}\delta + \Psi\right)$$

$$\theta = i\vec{k}.\vec{v}$$

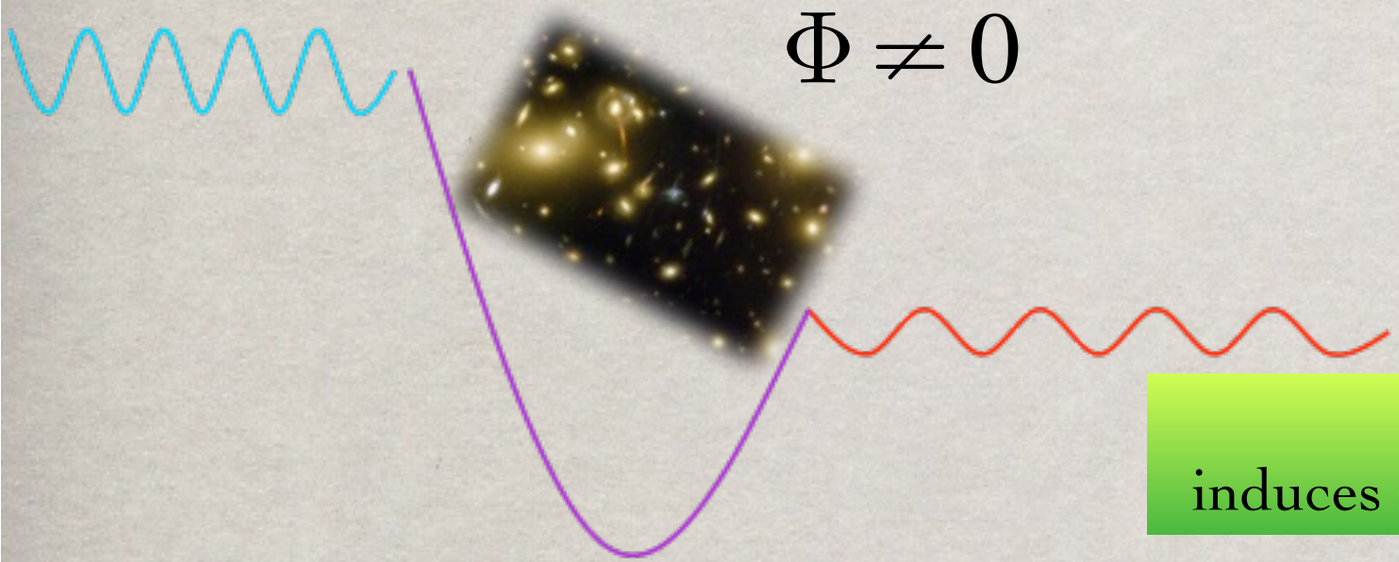
$$b \equiv \frac{\delta_g}{\delta_m}$$

$$P_g(k) = b^2 P_m(k)$$

$$\boxed{\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m - \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_m = 0}$$


1 Gpc/h
Millennium Simulation
10.077.696.000 particles
(z = 0)

ISW effect



$$\dot{\Phi} \neq 0$$

$$\frac{\Delta T(\hat{n})}{\bar{T}_0} = -\frac{2}{c^2}\int_0^{t_L} \dot{\Phi}(t,\hat{n})dt$$

Gravitational potential change
induces secondary CMB temperature fluctuation

- Linear regime:Integrated Sachs-Wolfe effect(ISW), (1967)
- Non-linear regime:Rees-Sciama effect (RS), (1968)
- ISW & RS effect capture the dynamics of the universe

ISW effect

$$\nabla^2\Phi = 4\pi G a^2 \rho\delta \quad \rightarrow \Phi \propto \frac{\delta}{a}$$

$$\delta \equiv \frac{\delta\rho}{\rho}$$

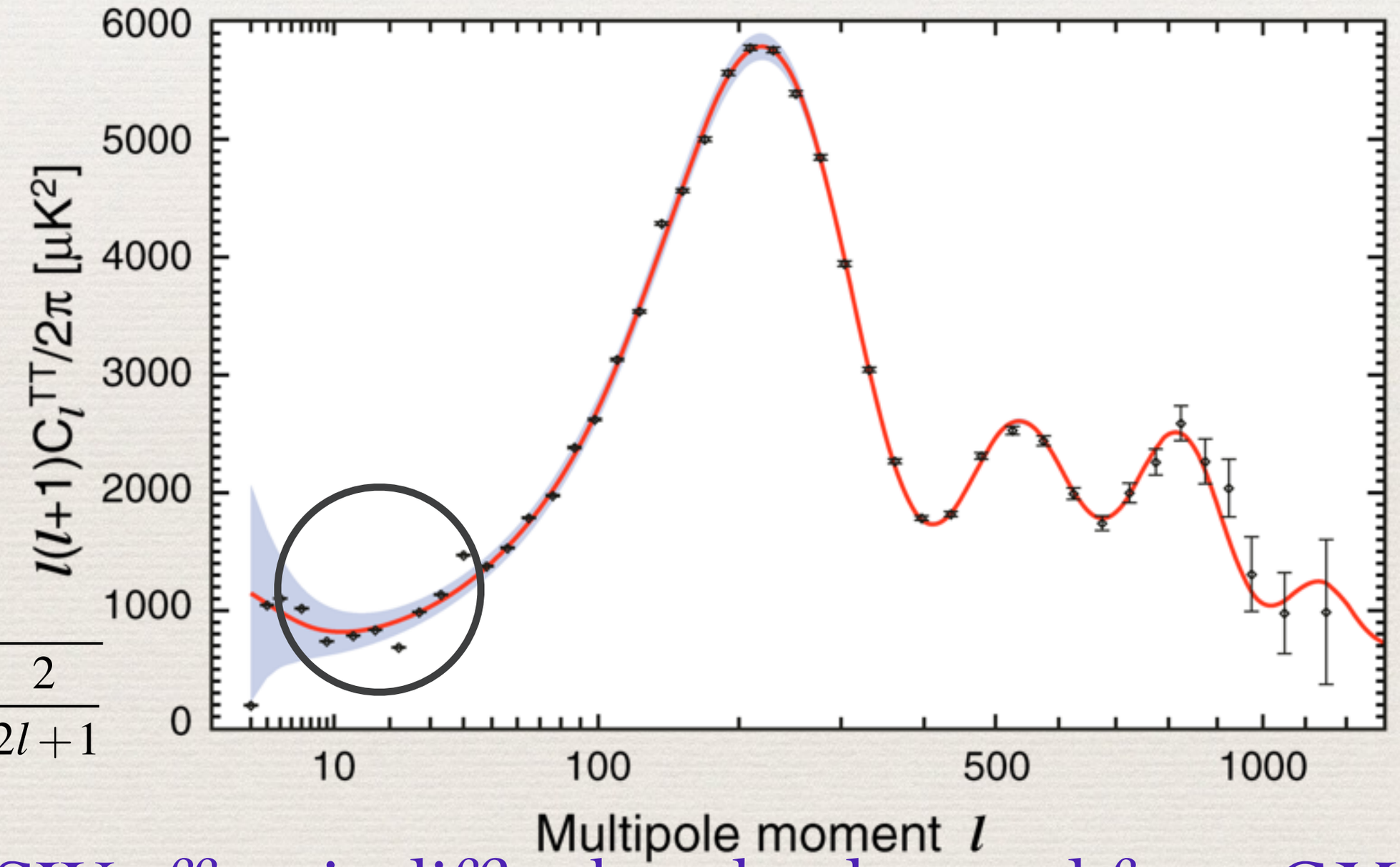
$$\delta \propto a \quad \dot{\Phi} = 0 \longrightarrow \text{Matter domination}$$

No effect in matter epoch

What can the ISW do for us?

- Independent evidence for Dark Energy
  - Matter dominated universe in trouble
- Direct probe of the evolution of structures
  - Do the gravitational potentials grow or decay?
  - Constrain modified gravity models?
- Sensitive on the largest scales (horizon)
  - Measure dark energy clustering

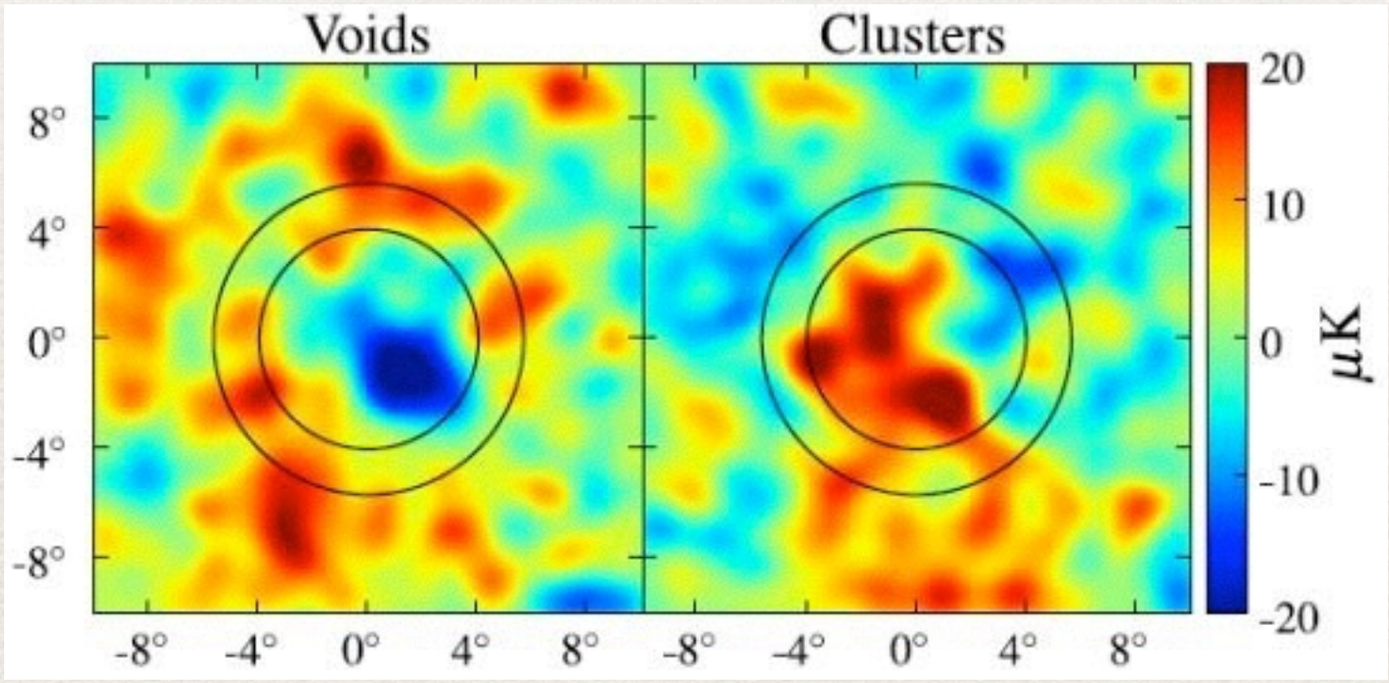
Imprint on the CMB power spectrum



$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}}$$

The ISW effect is difficult to be detected from CMB power spectrum due to Cosmic Variance problem

ISW evidence for DE

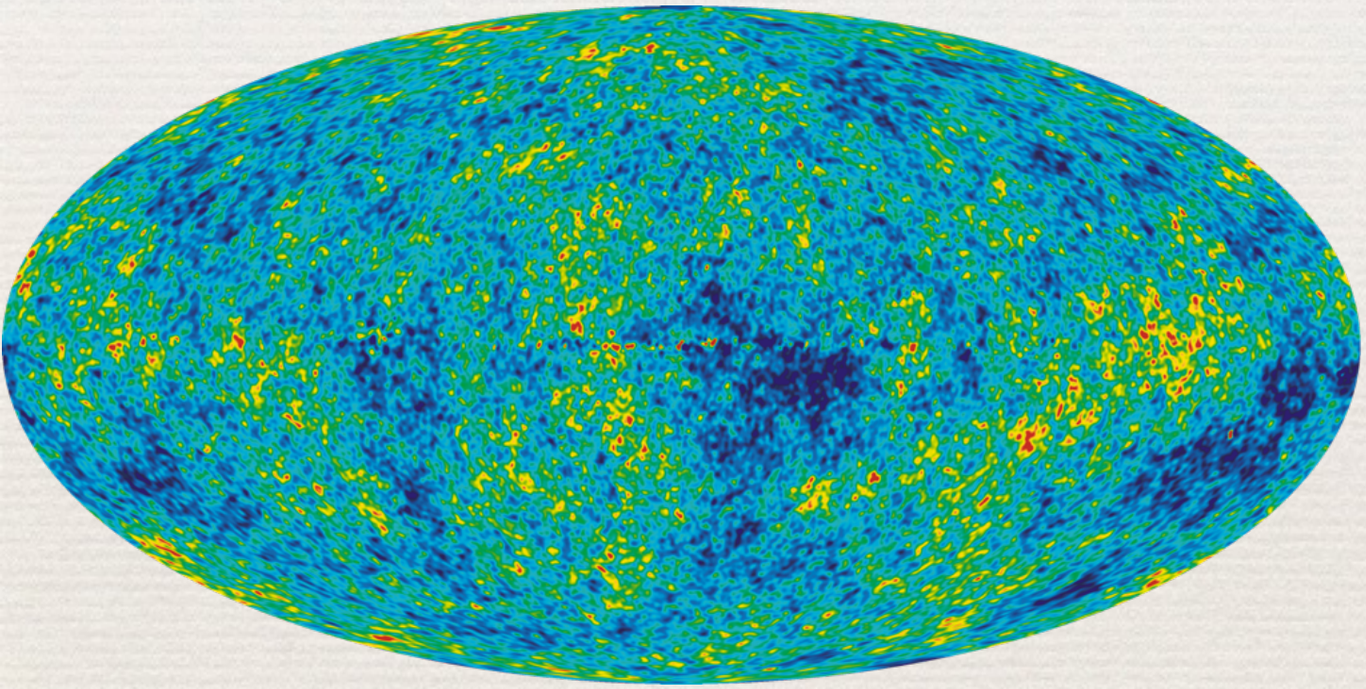


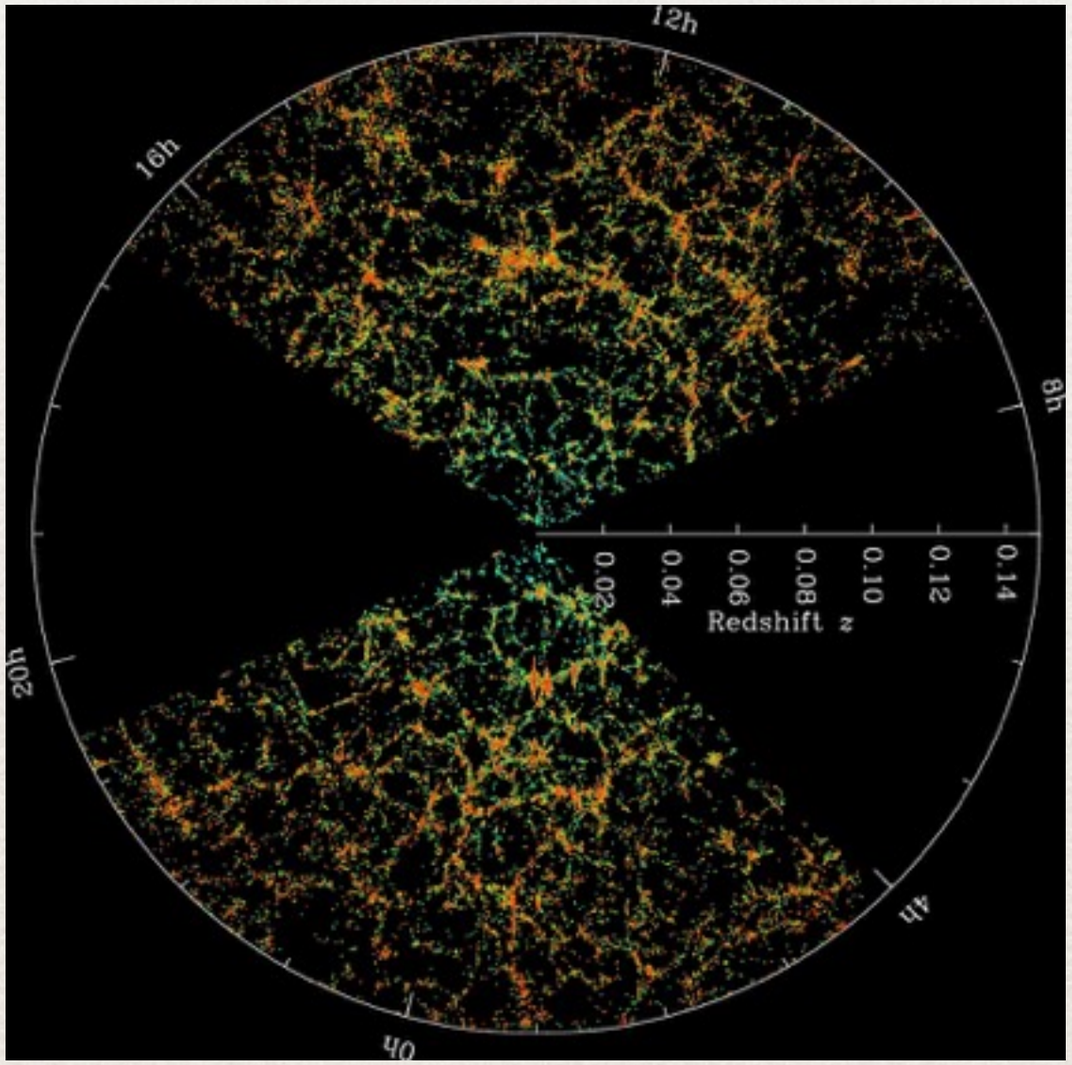
The greater temperature of superclusters (right) compared with supervoids (left) tells Szapudi and his colleagues that the superclusters are being stretched by dark energy.
(Credit: István Szapudi *et al*.)

Cross Correlation

- A signature of the ISW effect is a non-zero cross-correlation function between the galaxy density (the number of galaxies per square degree) and the temperature of the CMB, because superclusters gently heat photons, while supervoids gently cool them

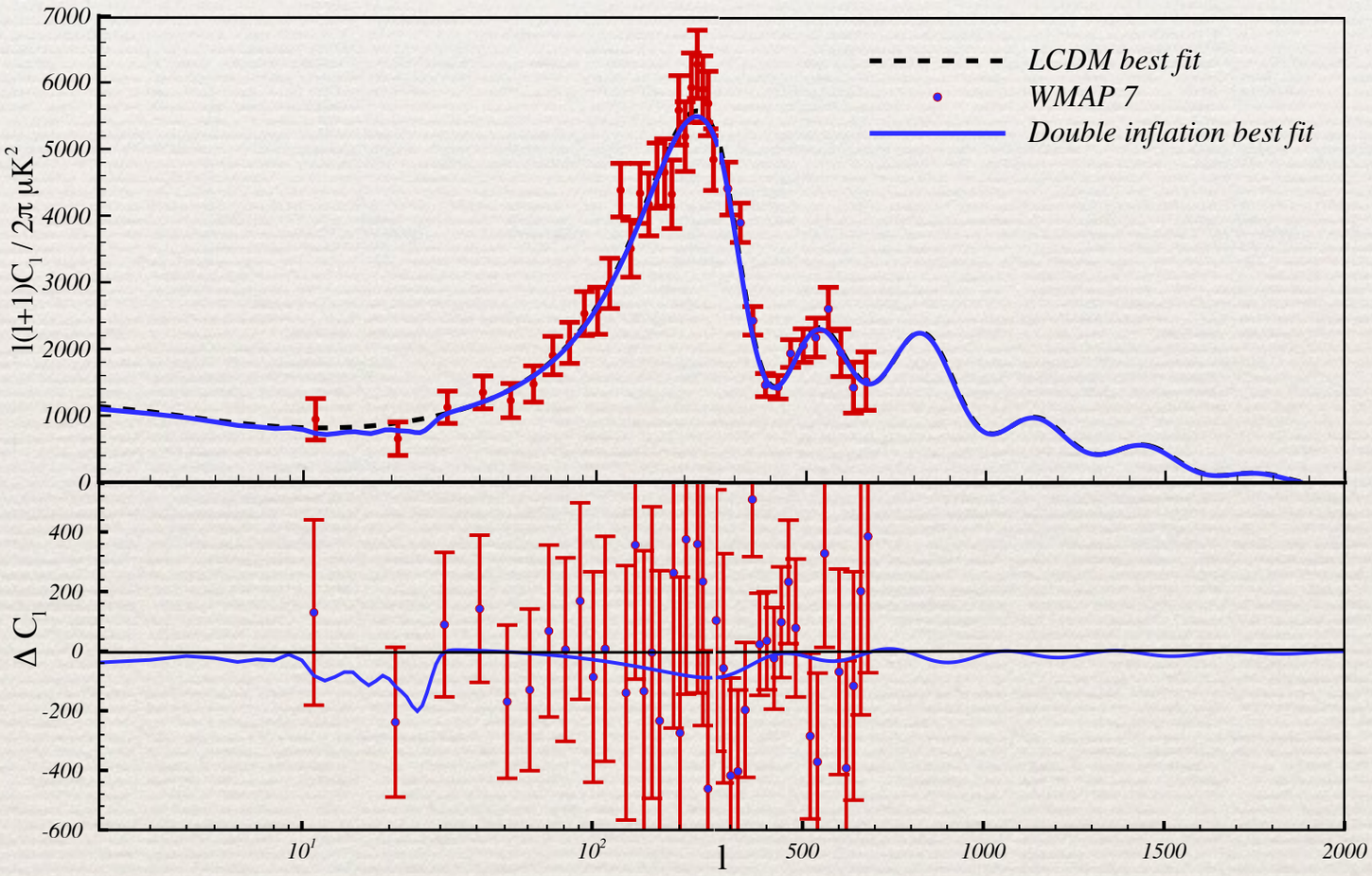
Cross Correlation



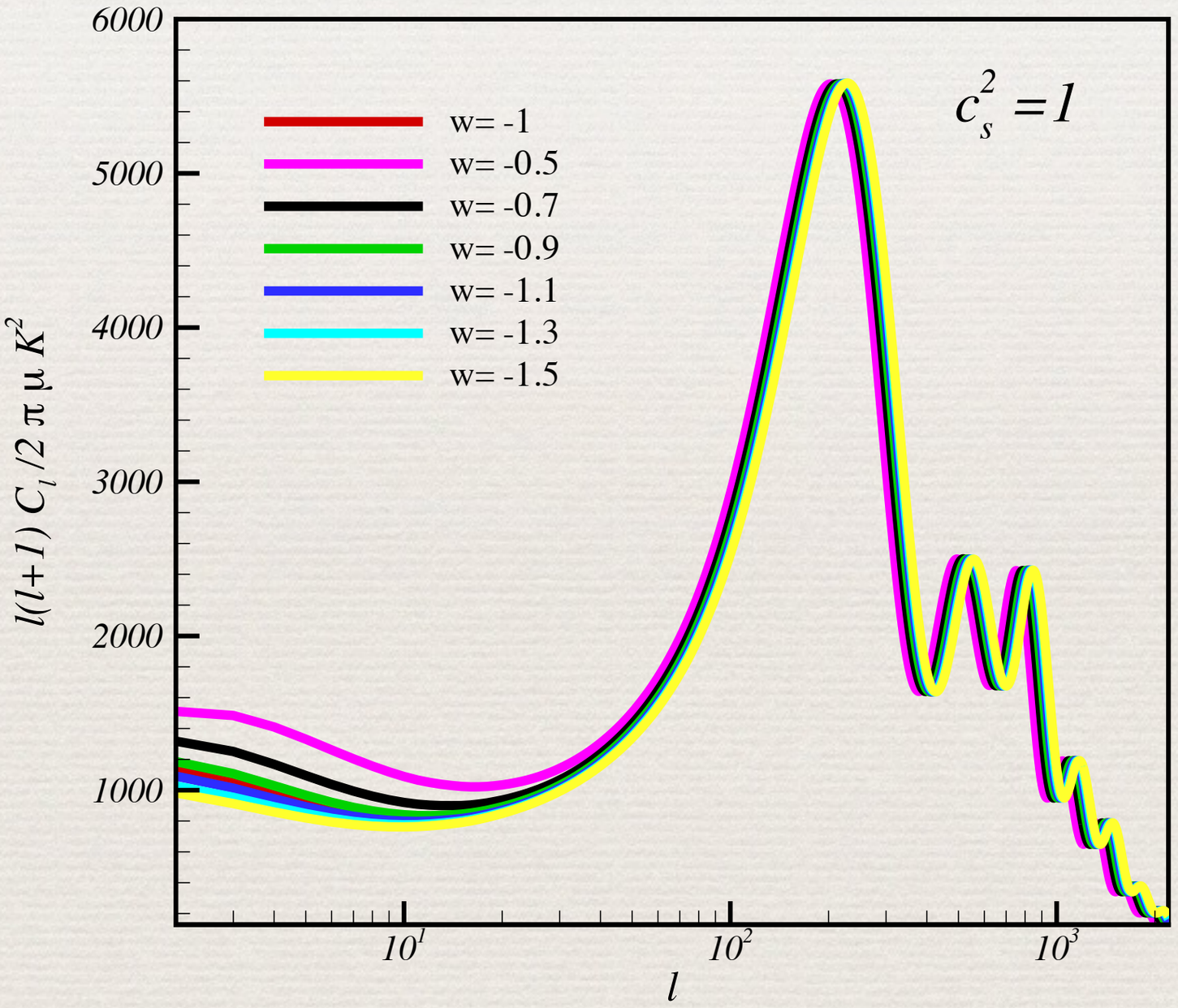


$$C_l^{gal-ISW} = \langle \delta_{lm}^{gal} \Theta_{lm}^{ISW} \rangle$$

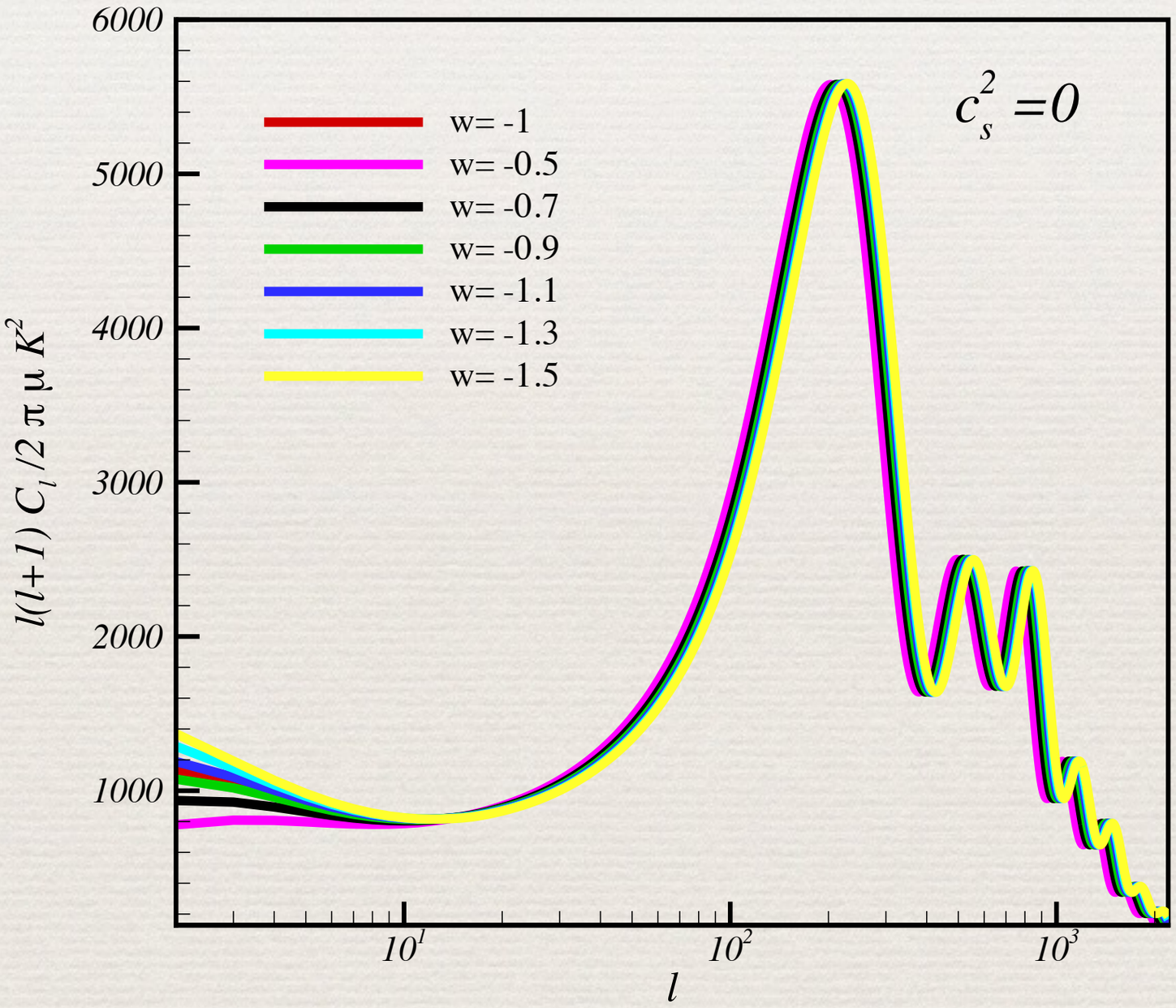
Dark Energy Clustering



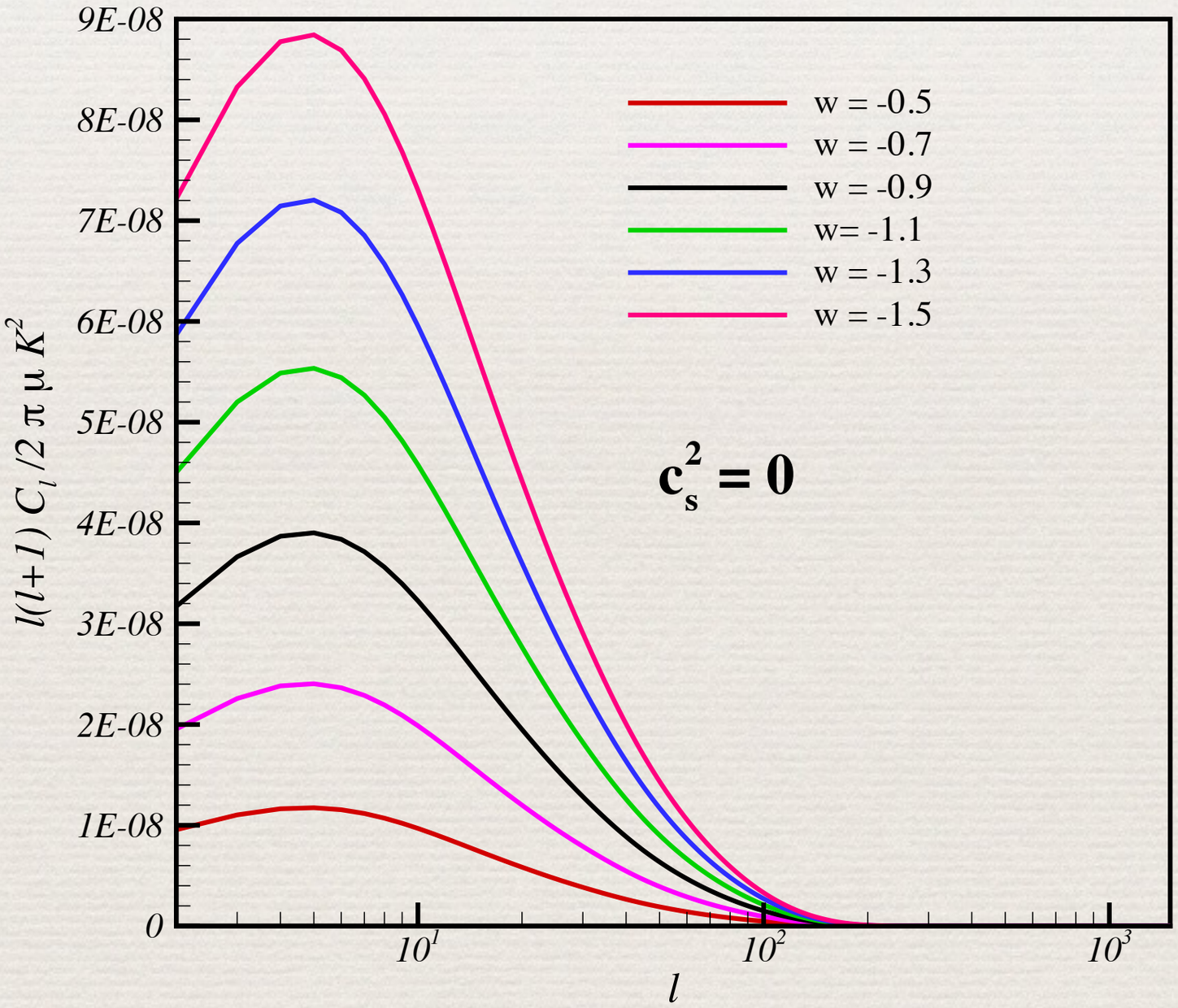
CMB power spectrum



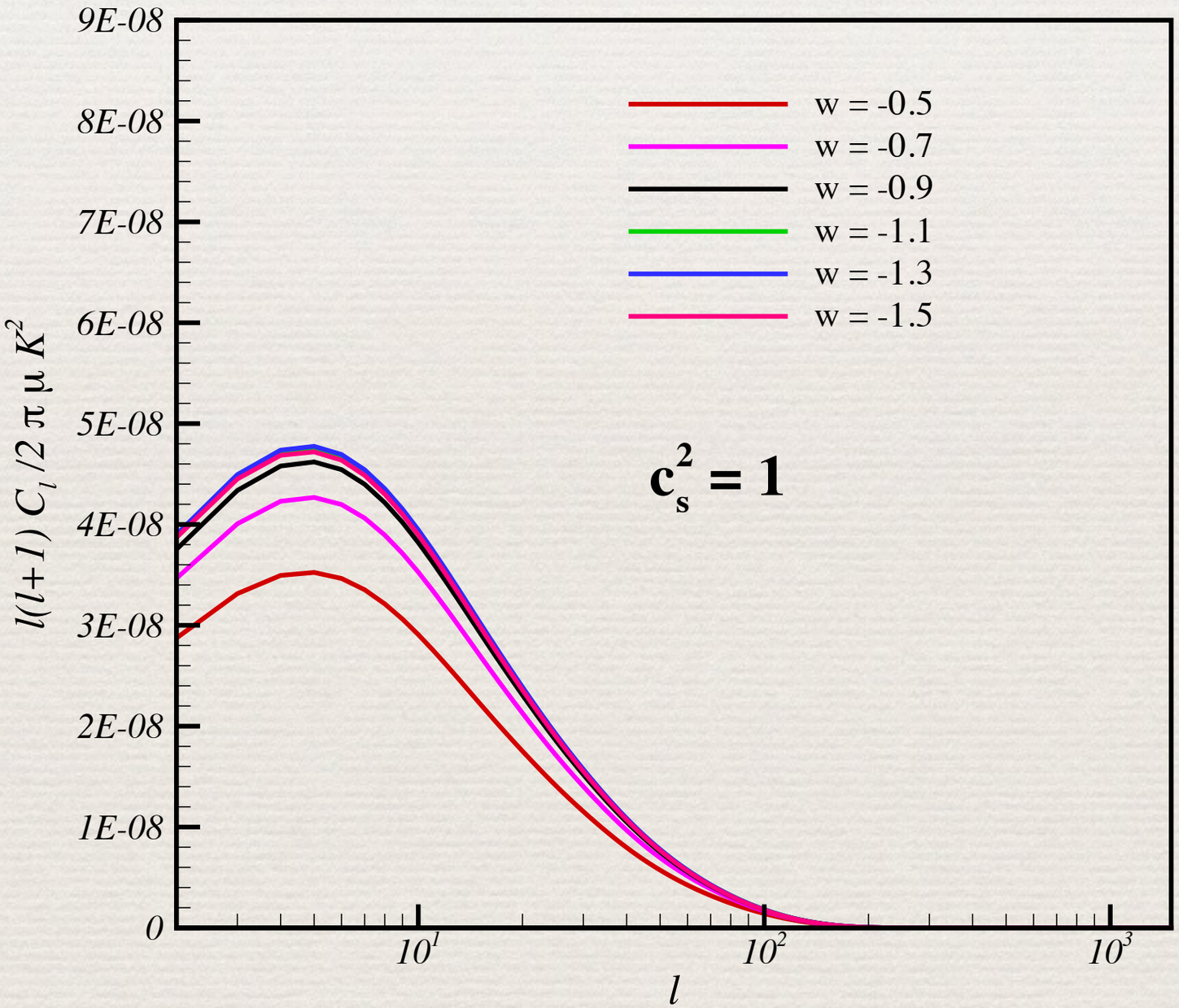
CMB power spectrum



ISW spectrum



ISW spectrum



Thank You