An Introduction to *CosmoMC*

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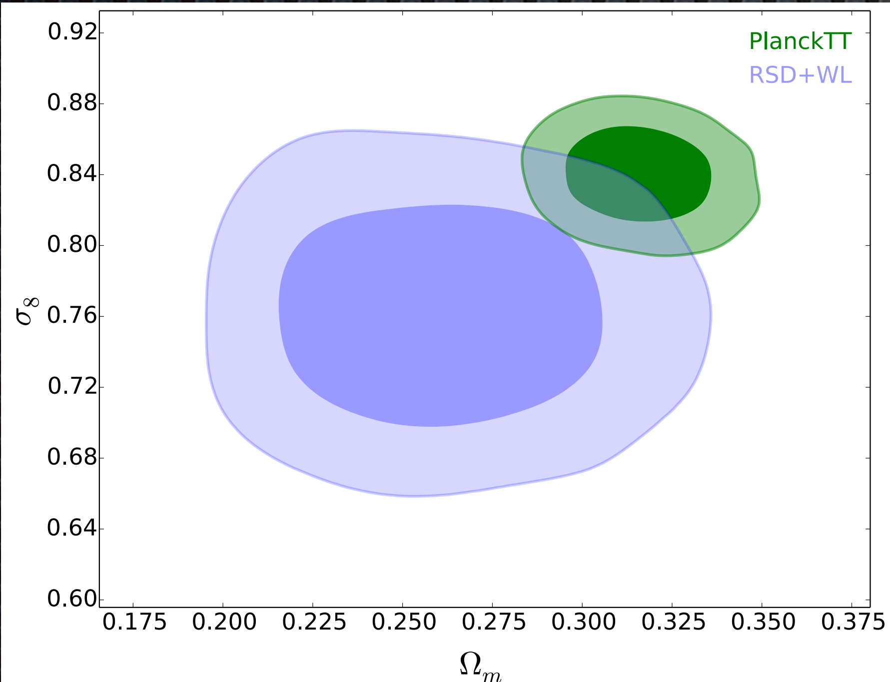
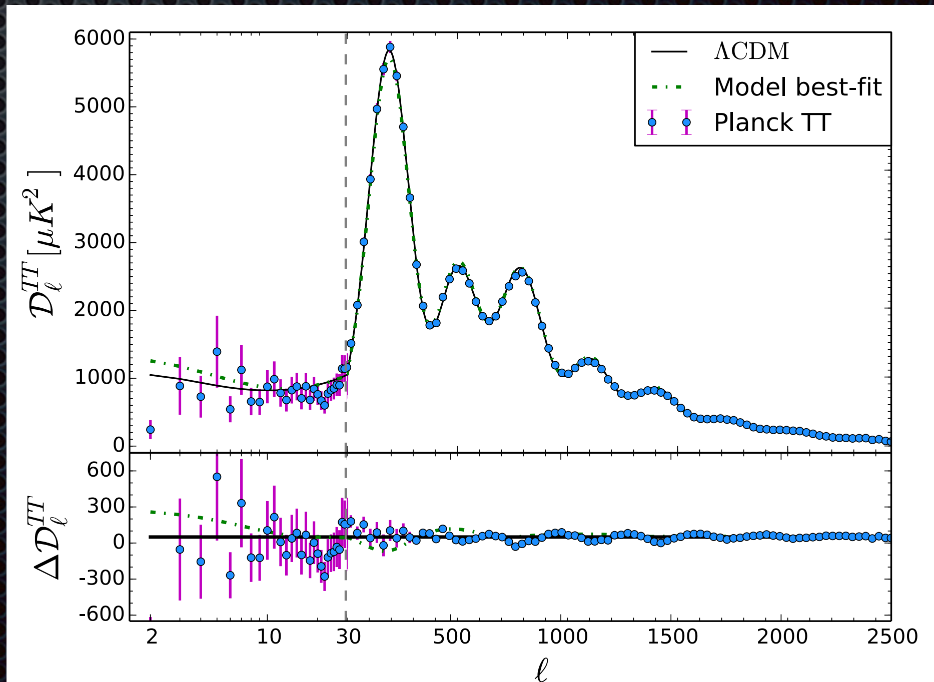


SUT, Fall 2017



Outline

- Statistical methods
- Degeneracies and marginalization
- 6 parameter based model
- Cosmological datasets
- CosmoMC
- MCMC
- Installation

Standard Model

- Standard model of cosmology is current simplest framework to describe the cosmological observations.
- The standard cosmological model is ΛCDM model. In this model, the Universe contains dark matter (and baryonic) , and the accelerated expansion rate is due to a cosmological constant (Λ).
- Also the gravity is defined by FLRW metric.

Standard Model

- There are “6” parameters in the ‘base’ ΛCDM model:

$$\left\{\Omega_b h^2, \Omega_c h^2, H_0, \tau_{\rm re}, A_s, n_s\right\}$$

- Physical density of baryonic matter $\Omega_b h^2$
- Physical density of cold dark matter $\Omega_c h^2$
- The local expansion rate H_0
- The optical depth to reionization $\tau_{\rm re}$
- Amplitude of primordial scalar power spectrum $A_{\rm S}$
- Spectral index of scalar power spectrum $n_{\rm S}$

Beyond the Standard Model

- There are some additional parameters not included in base model.
- The curvature of the Universe Ω_K
- The amount of matter in the form of massive neutrinos $\sum m_\nu$
- The effective number of relativistic species at recombination N_{eff}
- The equation of state of dark energy (and its time dependence) w_0, w_a
- The tensor to scalar power spectrum ratio r
- Number of e-folds N
- Non-Gaussianity f_{NL}
- Running of spectral index a
- Modified gravity parameters

Cosmological Observations

- Cosmic Microwave Background (temperature and polarization)
- Matter power spectrum (clusters, weak lensing, lyman alpha)
- Standard markers (candles, rulers, clocks,…)

Cosmological Datasets

- **CMB**. Angular power spectrum of temperature and polarization. Latest measurements by Planck satellite.
- **BAO**. Cosmic distance scale measurements using Baryon Acoustic Oscillations. Latest is BOSS.
- **SN**. Type-Ia Supernovae. Distance modulus from the luminosity distance derived by the explosion of SNIa. Latest compilation by Joint Light Curve analysis (JLA).

Cosmological Datasets

- **P(k)**. Measured by clustering of the galaxy distribution or distribution of Lyman- Alpha Forest.
- **H0**. Direct measurements of the expansion rate in the local Universe using cepheids, SN host galaxies, and water maser distances.
- **RSD**. Redshift space distortions. Measuring the clustering of cosmic tracers such as galaxies and Ly-alpha forest in redshift space it is possible to derive structure growth and test GR. Every galaxy survey has provided a measurement.
- Others: Weak-Lensing, SZ, …

CAMB

Code for Anisotropies in the Microwave Background.

- Boltzmann code (linear perturbation theory) to solve a realization for a given cosmological model and generate observables to be fitted with data, e.g., the expansion rate of the universe, anisotropy power spectrum of perturbations of the cosmic microwave background and the matter power spectrum.

- CAMB is written in Fortran and based on the CMBFast. The most recent version uses Fortran 90 specifications.

- It returns quantities like: CMB power spectrum, matter power spectrum, distance-redshift relation and etc.

- It has a large comunity and several extensions made by different people.

CAMB

Code for Anisotropies in the Microwave Background.

- Includes: Halofit; scalar, vector and tensor modes; polarization; lensed CMB
and lensing potential

- Internal parallelization of loops; support for adiabatic or isorcurvature initial conditions; estimates bispectrum; controlable accuracy

- CAMB has online version:
https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

- CAMB source :
http://camb.info

Einstein-Boltzmann Equations.

Liouville term ⟷ **Collision term**

Einstein+Boltzmann equ.

1) $\dot{\Theta}_T + ik\mu\Theta_T + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi]$

2) $\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}[-\Theta_P + \frac{1}{2}(1-\mathcal{P}_2(\mu))\Pi]$

$\Pi = \Theta_{T2} + \Theta_{P2} + \Theta_{P0}$

3) $\dot{\mathcal{N}} + ik\mu\mathcal{N} + \dot{\Phi} + ik\mu\Psi = 0$

Photon/Neutrinos

EB equs. with fluid approximation (Euiler+continuity)

4) $\dot{\delta}_c = -ikv_c - 3\dot{\Phi}$

5) $\dot{\delta}_b = -ikv_b - 3\dot{\Phi}$

6) $\dot{v}_c = -Hv_c - ik\Psi$

7) $\dot{v}_b = -Hv_b - ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]$, $\quad \frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$

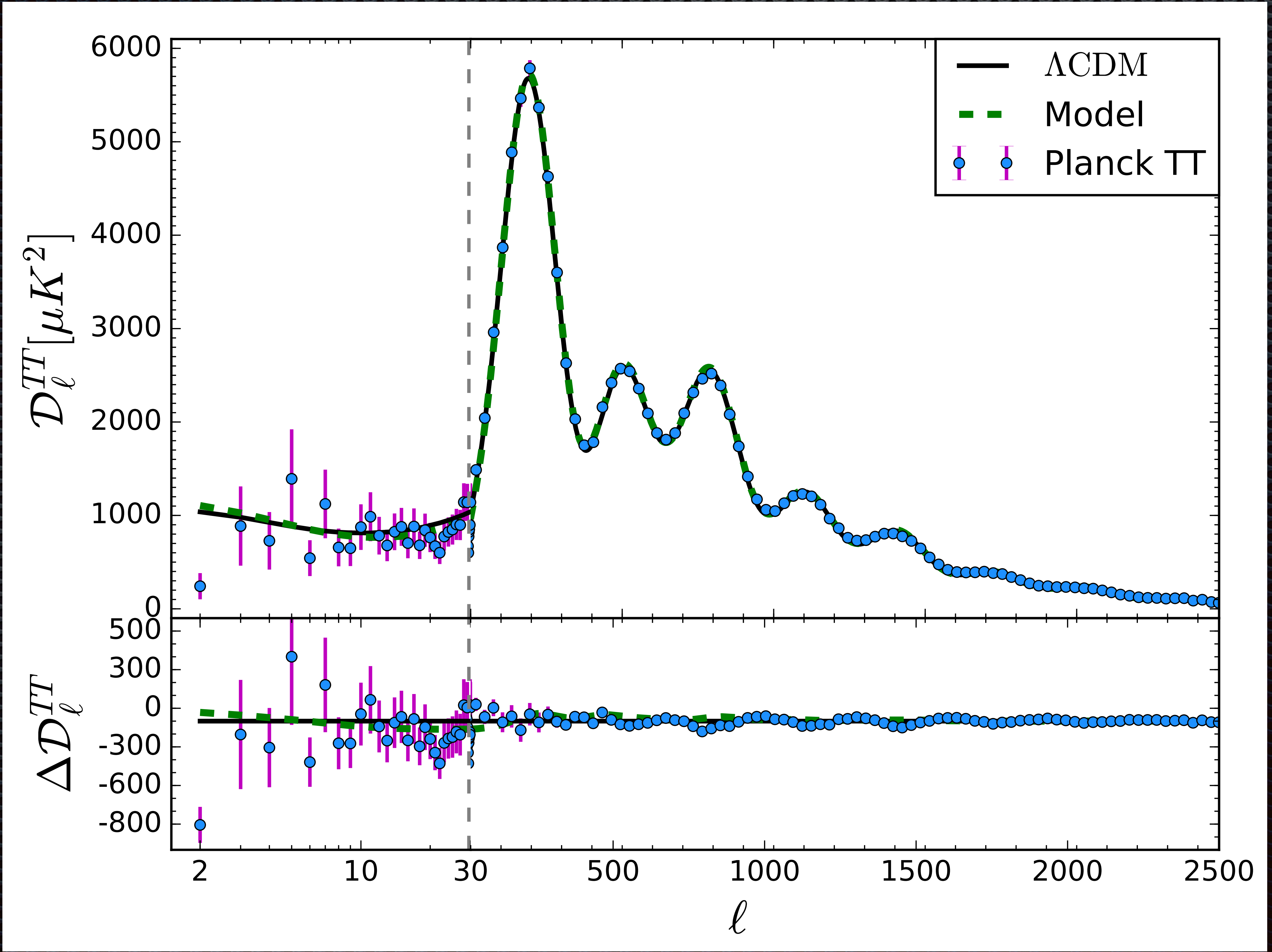
Baryon/CDM

CAMB

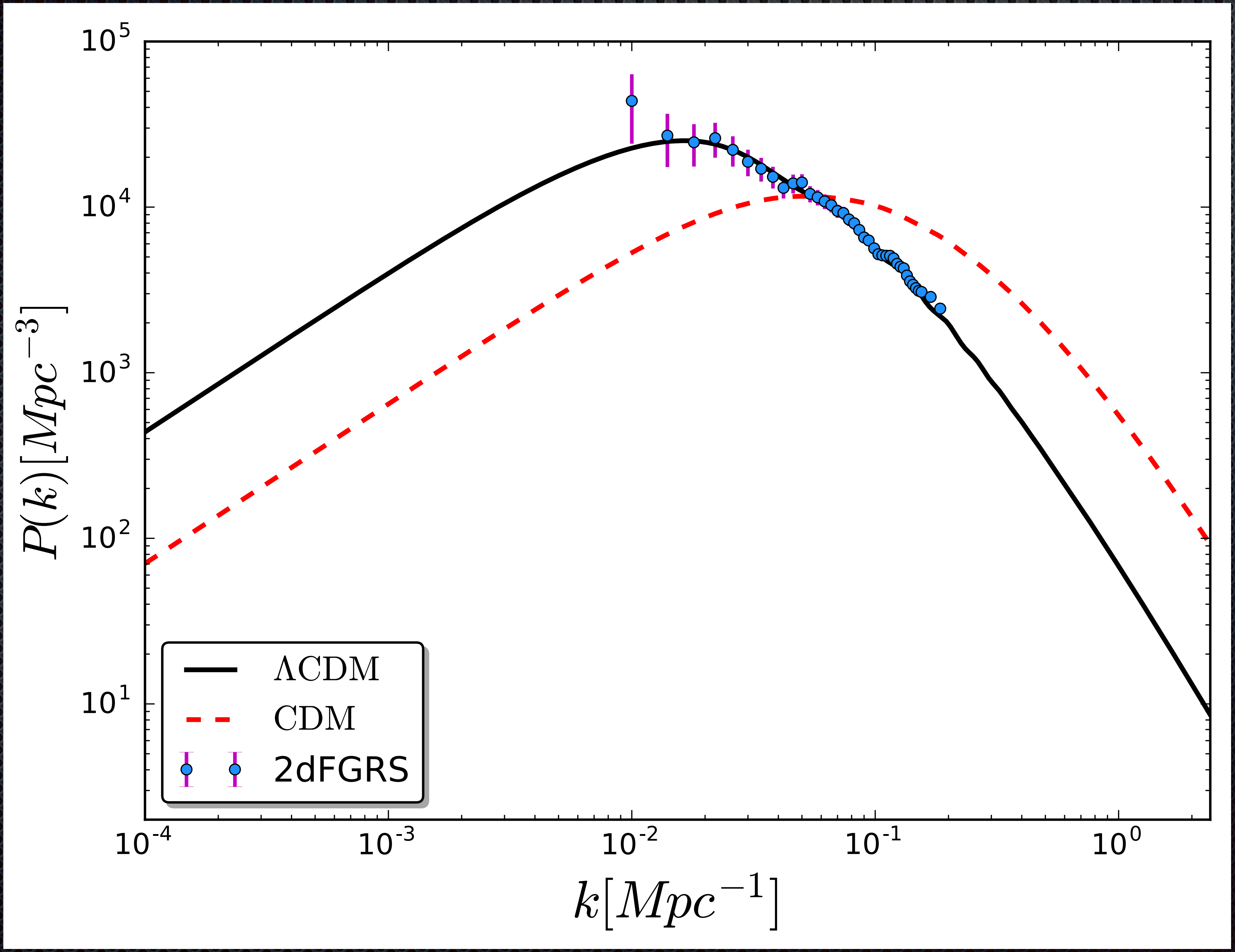
Code for Anisotropies in the Microwave Background.

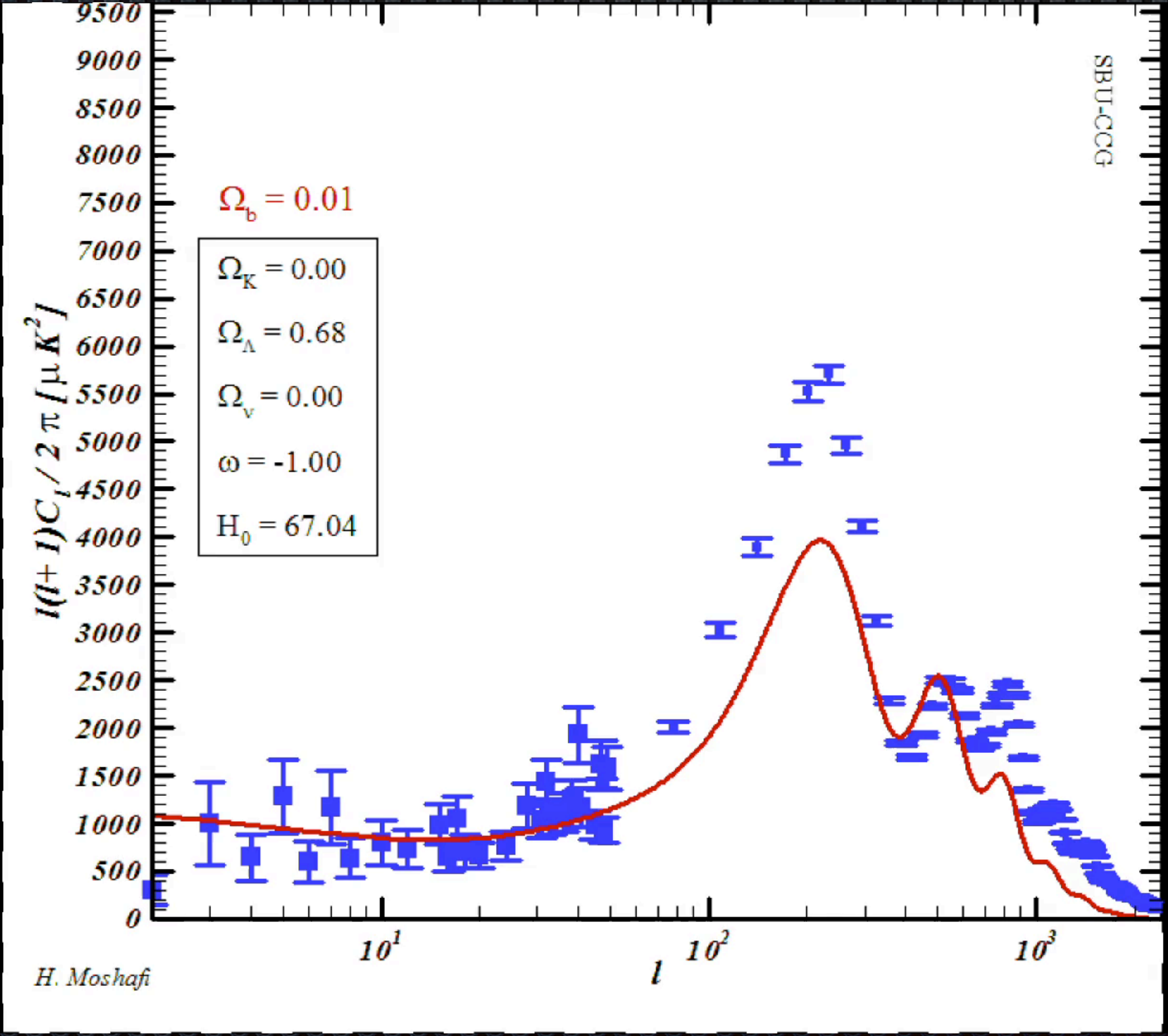
- Outputs:
  - Spectrum of the cosmic microwave background anisotropies:
    - root_scalCls.dat: {l, CTT, CEE, CTE, [Cphiphi, CphiT]}
    - root_lensedCls.dat: {l, CTT, CEE, CBB, CTE}
    - root_lenspotentialCls.dat: {l, CTT, CEE, CBB, CTE, Cdd, CdT, CdE}
    - root_tensCls.dat: {l, CTT, CEE, CBB, CTE}
- Matter power spectrum : root_matterpower.dat : {k, Pk}
- Transfer function for all particle perturbations: root_transfer_out.dat: {k/h, Delta_CDM/k^2, Delta_b/k^2, Delta_r/k^2, Delta_nu/k^2, Delta_tot/k^2}
- CAMB has online version: **https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm**
- CAMB source : **http://camb.info**

Temperature anisotropies power spectrum

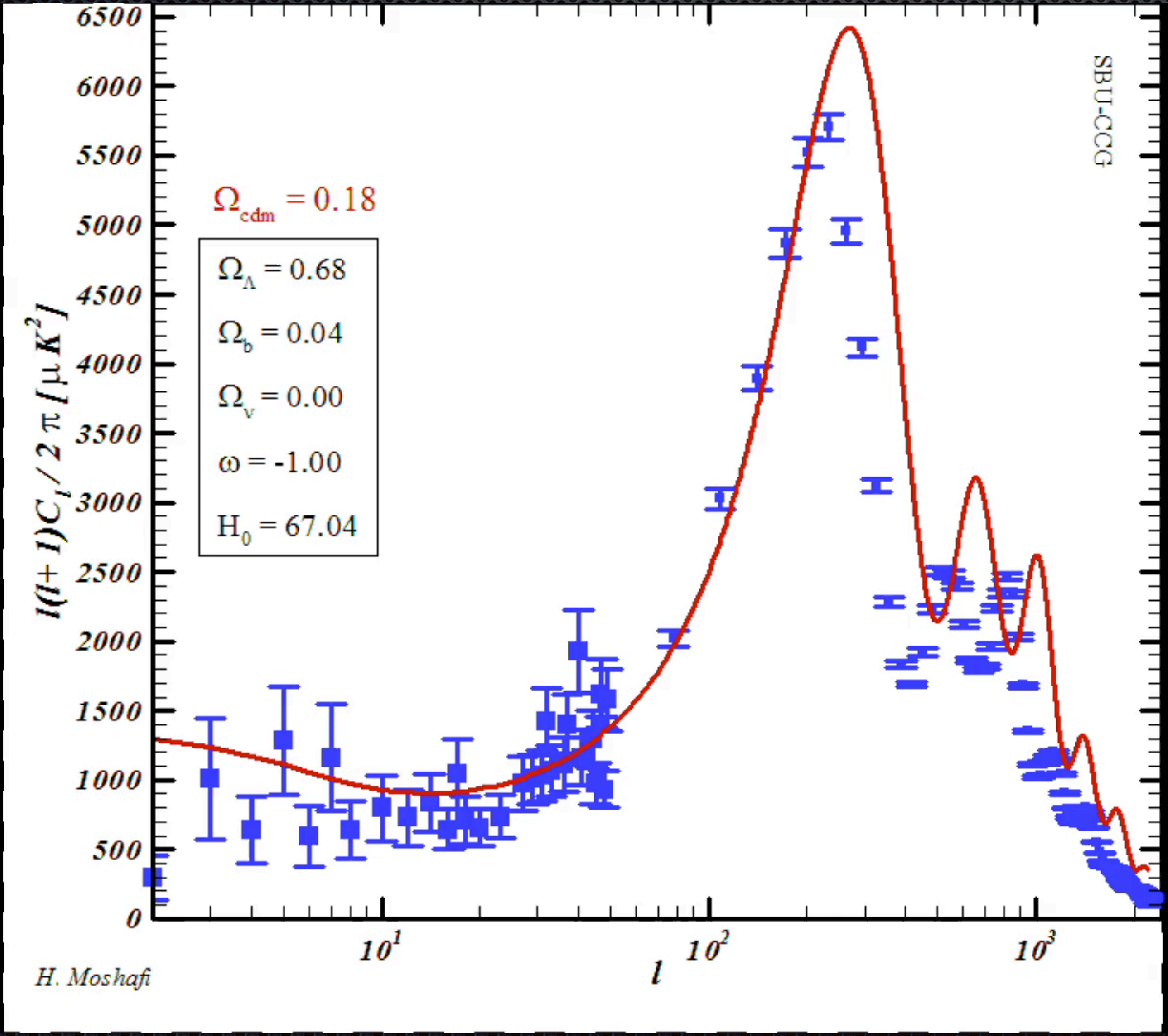


Matter power spectrum

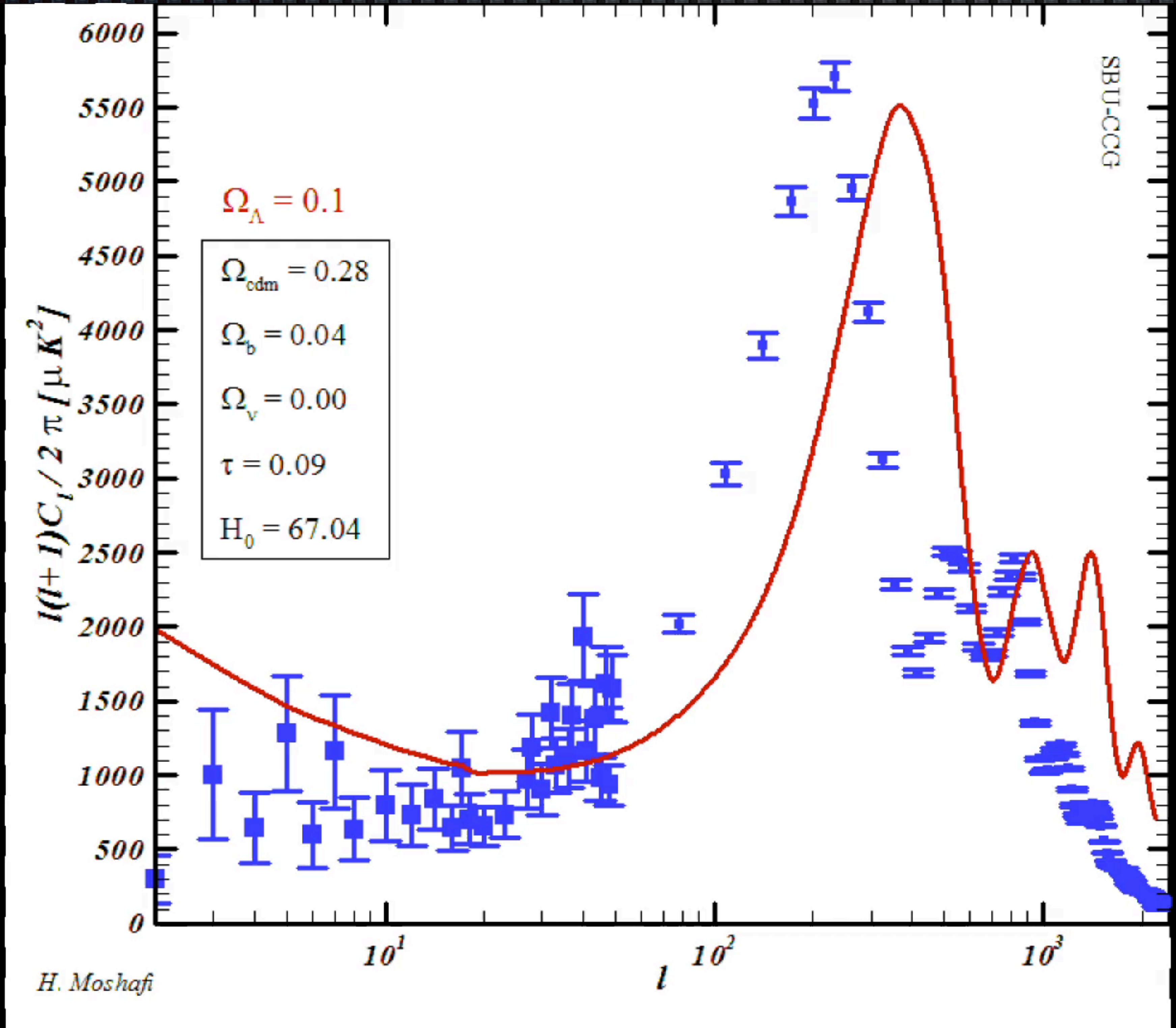




$\Omega_b = 0.01$
$\Omega_K = 0.00$
$\Omega_\Lambda = 0.68$
$\Omega_\nu = 0.00$
$\omega = -1.00$
$H_0 = 67.04$
$l(l+1)C_l / 2\pi\ [\mu K^2]$
l
9500
9000
8500
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
10^1
10^2
10^3
SBU-CCG
H. Moshafi



6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
l(l+ 1)C_l / 2 π [μ K^2]
Ω_cdm = 0.18
Ω_Λ = 0.68
Ω_b = 0.04
Ω_ν = 0.00
ω = -1.00
H_0 = 67.04
10^1
10^2
10^3
l
H. Moshafi
SBU-CCG



6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
$l(l+1)C_l/2\pi\,[\mu K^2]$
$\Omega_\Lambda = 0.1$
$\Omega_{cdm} = 0.28$
$\Omega_b = 0.04$
$\Omega_\nu = 0.00$
$\tau = 0.09$
$H_0 = 67.04$
SBU-CCG
10^1
10^2
10^3
l
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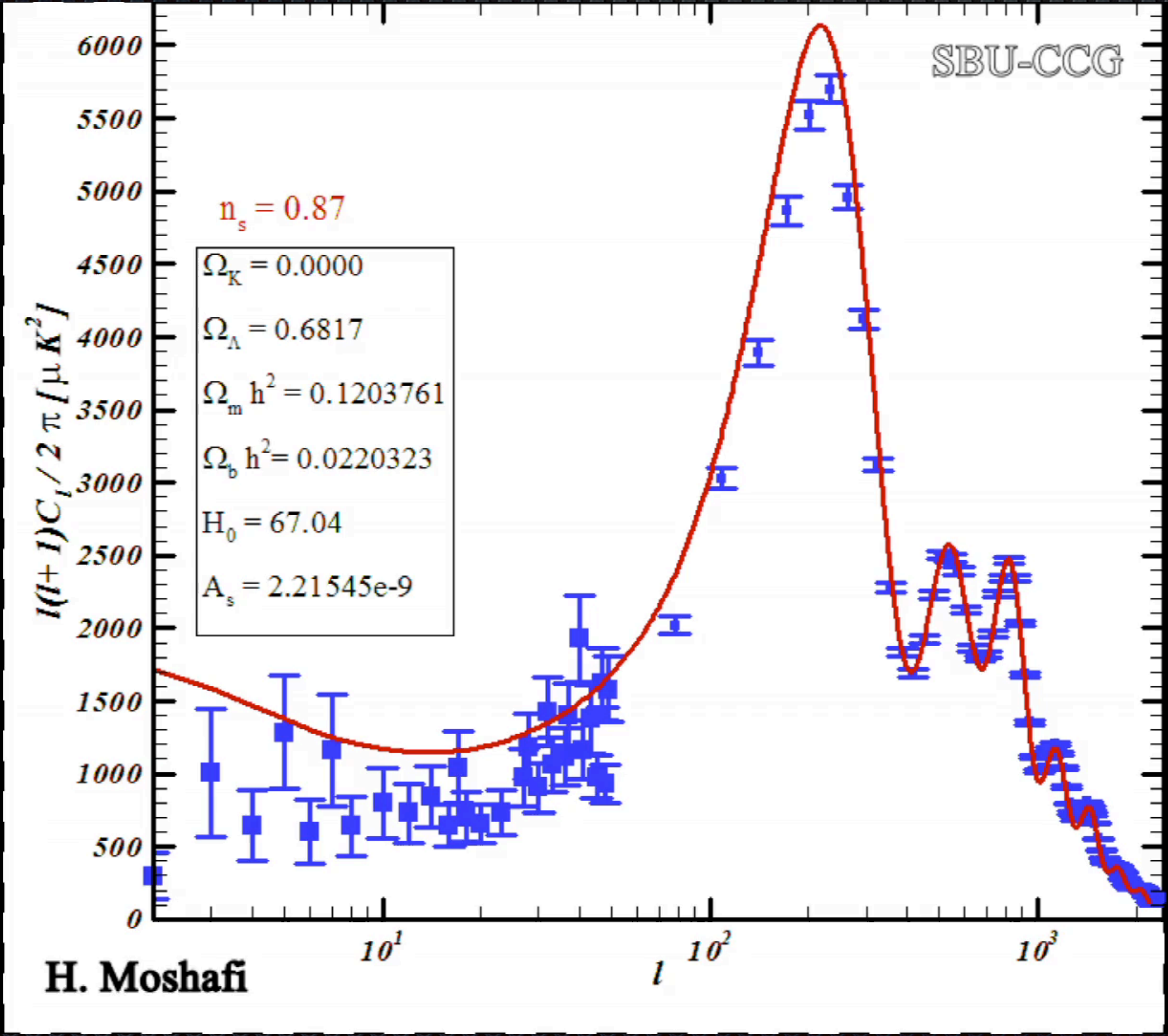

SBU-CCG
$n_s = 0.87$
$\Omega_K = 0.0000$
$\Omega_\Lambda = 0.6817$
$\Omega_m h^2 = 0.1203761$
$\Omega_b h^2 = 0.0220323$
$H_0 = 67.04$
$A_s = 2.21545e-9$
$l(l+1)C_l / 2\pi \, [\mu K^2]$
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
10^1
10^2
10^3
l
H. Moshafi

CosmoMC

- **Cosmo**logical **M**onte **C**arlo is a parameter *sampling* code, implements the Metropolis- Hastings algorithm to sample a given parameter space.
- It is bundled up with likelihood codes from the most recent data such as CMB likelihood from Planck, BAO likelihood from BOSS or SN likelihood from JLA.
- It is written in Fortran 90.
- GetDist package analyzes Markov chains and produces parameter tables and plots.
- Source code is available at: **http://cosmologist.info/cosmomc/**

Markov Chain Monte Carlo (MCMC)

- It is a **random walk** to effectively sample a multi-dimensional parameter space (when the number of dimensions is large, sampling a grid is expensive)

- We use the **Metropolis-Hastings** algorithm:
  if a step puts you in a place with more probability -> ACCEPT
  if not -> ACCEPT with probability=likelihood ratio=P_{new}/P_{old})
  otherwise, REJECT (try a different move from previous step)

- CosmoMC parallels MCMC processes. Independent, but communicating, MCMC going by different paths to reach the same best fit region with velocity and precision.

χ^2 and Likelihood

- The measurements in cosmological datasets are translated to **likelihoods**. The total likelihood, assuming the measurements of the experiments are not correlated, is the product of individual likelihoods.

- However in practice, CosmoMC uses the log of the likelihood (with opposite sign), which is closely related to the value of the χ^2 distribution. These are related by the equation:

$$\mathcal{L} \propto e^{-\frac{\chi^2}{2}}$$

- Where the χ^2 for simple cases is just (measurement-theory)^2/error^2, or more generally $\chi^2 = (\mathbf{D}-\mathbf{T})^t\,\mathbf{C}^{-1}\,(\mathbf{D}-\mathbf{T})$ where **D** is a vector of data measurements and **T** is the theory vector generated at each MCMC step. **C** is the **covariance matrix** whose elements are the covariances between parameters $\mathbf{C}=\{\sigma_{\theta i,\theta j}\}$, so that the diagonal elements are $\sigma^2_{\theta i}$ and the off-diagonal are $\sigma_{\theta i \theta j} = \rho_{ij}\,\sigma_{\theta i}\,\sigma_{\theta j}$

MCMC chains

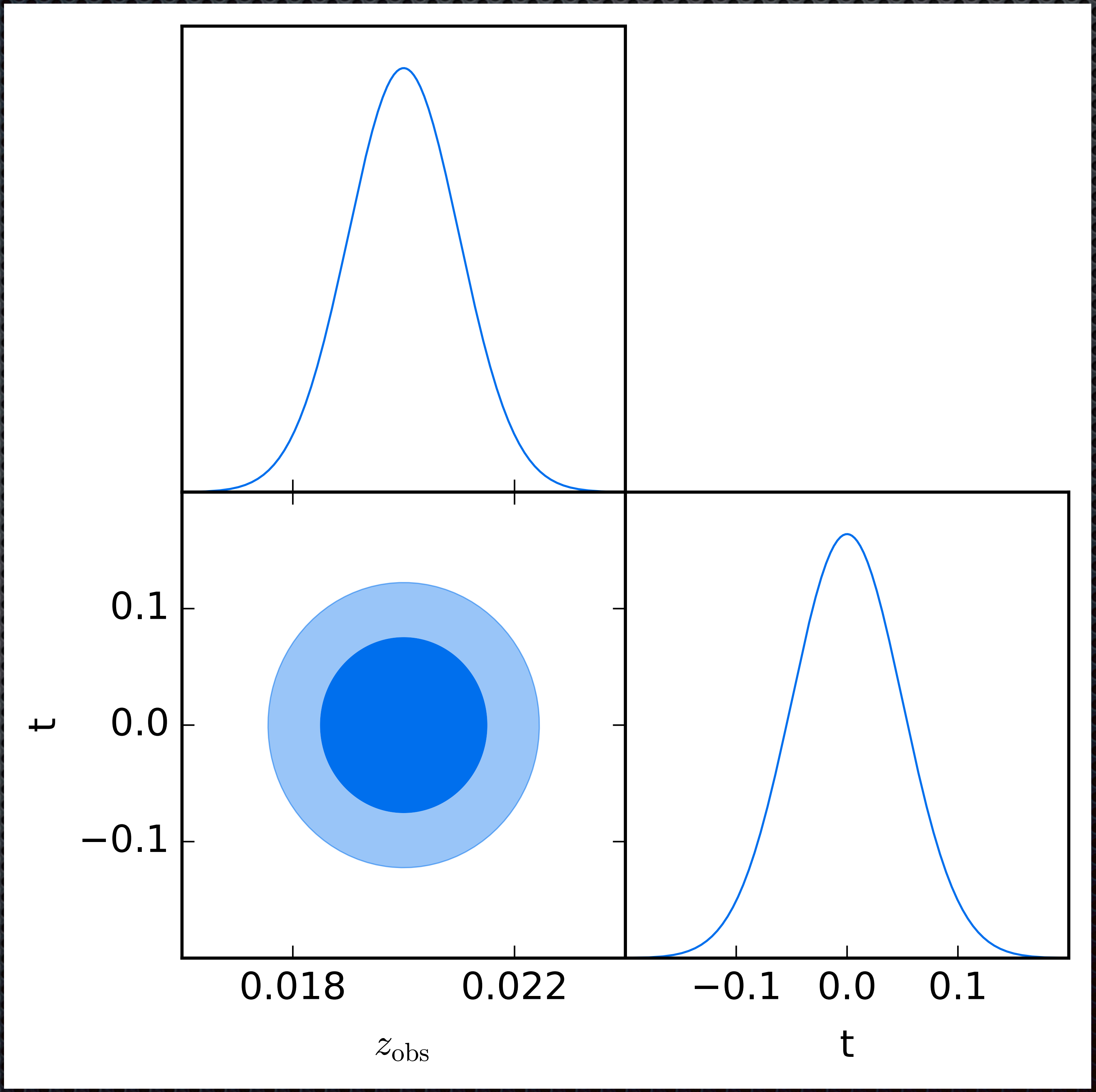
number of steps -log(L) **Model parameters**

0.1000000E+01	0.4759569E+03	0.2213998E-01	0.1172434E+00	0.1041655E+01	0.8053226E-01	0.3099537E+01	0.9640315E+00	0.6839050E+02	0.7006185E+00	0.2993815E+00
0.1400283E+00	0.6451439E-03	0.9576621E-01	0.8254270E+00	0.4516387E+00	0.6105692E+00	0.9981154E+00	0.2483167E+01	0.1019135E+02	0.2218767E+01	0.1888697E+01
0.1254866E+04	0.5805181E+04	0.2553392E+04	0.8182748E+03	0.2300445E+03	0.9640315E+00	0.2452879E+00	0.2466140E+00	0.2634928E+01	0.1378663E+02	0.1089967E+04
0.1453257E+03	0.1041870E+01	0.1394855E+02	0.1059208E+04	0.1480835E+03	0.1396480E+00	0.1612821E+00	0.3330813E+04	0.1016605E-01	0.8262200E+00	0.4563183E+00
0.3544231E-01	0.1844947E+03	0.1783464E+04	0.3073164E+01	0.3528807E+00	0.3724484E+00	0.9357578E+03	0.1590595E+02	0.2500000E+00	0.9516638E+03	
0.2000000E+01	0.4648023E+03	0.2220156E-01	0.1175487E+00	0.1041608E+01	0.8158817E-01	0.3098752E+01	0.9642759E+00	0.6831360E+02	0.6991579E+00	0.3008421E+00
0.1403954E+00	0.6451439E-03	0.9590916E-01	0.8259503E+00	0.4530261E+00	0.6117001E+00	0.9993103E+00	0.2484429E+01	0.1027374E+02	0.2217027E+01	0.1883235E+01
0.1250557E+04	0.5784877E+04	0.2545391E+04	0.8162917E+03	0.2296715E+03	0.9642759E+00	0.2453178E+00	0.2466441E+00	0.2623189E+01	0.1378339E+02	0.1089916E+04
0.1451980E+03	0.1041810E+01	0.1393709E+02	0.1059361E+04	0.1479332E+03	0.1398533E+00	0.1611777E+00	0.3339583E+04	0.1019281E-01	0.8247286E+00	0.4554918E+00
0.3541956E-01	0.1846745E+03	0.1783333E+04	0.3075932E+01	0.3528764E+00	0.3723092E+00	0.9137652E+03	0.1558944E+02	0.2500000E+00	0.9293547E+03	
0.8000000E+01	0.4622905E+03	0.2212092E-01	0.1172065E+00	0.1041523E+01	0.8378934E-01	0.3098017E+01	0.9666413E+00	0.6834290E+02	0.7003211E+00	0.2996789E+00
0.1399726E+00	0.6451439E-03	0.9566130E-01	0.8254475E+00	0.4518742E [illegible]	[illegible]	0.9984878E+00	0.2476061E+01	0.1048431E+02	0.2215397E+01	0.1873584E+01
0.1239266E+04	0.5744383E+04	0.2535189E+04	0.8134976E+03	0.2288368E [illegible]	[illegible]	0.2452784E+00	0.2466045E+00	0.2638579E+01	0.1379240E+02	0.1089989E+04
0.1453501E+03	0.1041736E+01	0.1395267E+02	0.1059170E+04	0.1481141E [illegible]	[illegible]	0.1612888E+00	0.3329476E+04	0.1016197E-01	0.8263077E+00	0.4563754E+00
0.3543580E-01	0.1844453E+03	0.1784270E+04	0.3073729E+01	0.3528447E+00	0.3723820E+00	0.9096629E+03	0.1466813E+02	0.2500000E+00	0.9243310E+03	
0.2000000E+01	0.4623016E+03	0.2211407E-01	0.1172279E+00	0.1041534E+01	0.8326129E-01	0.3098147E+01	0.9661860E+00	0.6833191E+02	0.7001935E+00	0.2998065E+00
0.1399872E+00	0.6451439E-03	0.9565590E-01	0.8254732E+00	0.4519845E+00	0.6108200E+00	0.9985992E+00	0.2477396E+01	0.1044159E+02	0.2215686E+01	0.1875808E+01
0.1241554E+04	0.5752319E+04	0.2537675E+04	0.8140731E+03	0.2289583E+03	0.9661860E+00	0.2452750E+00	0.2466011E+00	0.2639893E+01	0.1379298E+02	0.1089999E+04
0.1453497E+03	0.1041744E+01	0.1395253E+02	0.1059132E+04	0.1481183E+03	0.1395938E+00	0.1612987E+00	0.3329824E+04	0.1016304E-01	0.8262294E+00	0.4563400E+00
0.3543573E-01	0.1844495E+03	0.1784370E+04	0.3073971E+01	0.3528358E+00	0.3723609E+00	0.9094987E+03	0.1485438E+02	0.2500000E+00	0.9243531E+03	
0.2000000E+01	0.4613732E+03	0.2210341E-01	0.1172809E+00	0.1041563E+01	0.8433752E-01	0.3099491E+01	0.9677021E+00	0.6831360E+02	0.6999421E+00	0.3000579E+00
0.1400294E+00	0.6451439E-03	0.9565915E-01	0.8267670E+00	0.4528826E+00	0.6119055E+00	0.1000298E+01	0.2476488E+01	0.1054230E+02	0.2218666E+01	0.1874292E+01
0.1236779E+04	0.5737167E+04	0.2536528E+04	0.8143322E+03	0.2291435E+03	0.9677021E+00	0.2452698E+00	0.2465958E+00	0.2641943E+01	0.1379345E+02	0.1090018E+04
0.1453440E+03	0.1041774E+01	0.1395159E+02	0.1059132E+04	0.1481141E+03	0.1395880E+00	0.1613163E+00	0.3330835E+04	0.1016612E-01	0.8260345E+00	0.4562472E+00
0.3543439E-01	0.1844667E+03	0.1784480E+04	0.3074448E+01	0.3533498E+00	0.3728800E+00	0.9080494E+03	0.1444690E+02	0.2500000E+00	0.9224963E+03	
0.8000000E+01	0.4608853E+03	0.2207719E-01	0.1175800E+00	0.1041533E+01	0.8381279E-01	0.3099378E+01	0.9679784E+00	0.6817444E+02	0.6981288E+00	0.3018712E+00
0.1403023E+00	0.6451439E-03	0.9565031E-01	0.8279691E+00	0.4549095E+00	0.6137190E+00	0.1002775E+01	0.2479023E+01	0.1050873E+02	0.2218414E+01	0.1876047E+01
0.1235823E+04	0.5731581E+04	0.2538222E+04	0.8150410E+03	0.2293725E+03	0.9679784E+00	0.2452568E+00	0.2465828E+00	0.2646994E+01	0.1379833E+02	0.1090079E+04
0.1452855E+03	0.1041746E+01	0.1394635E+02	0.1059093E+04	0.1480631E+03	0.1396212E+00	0.1613385E+00	0.3337357E+04	0.1018602E-01	0.8247304E+00	0.4555861E+00

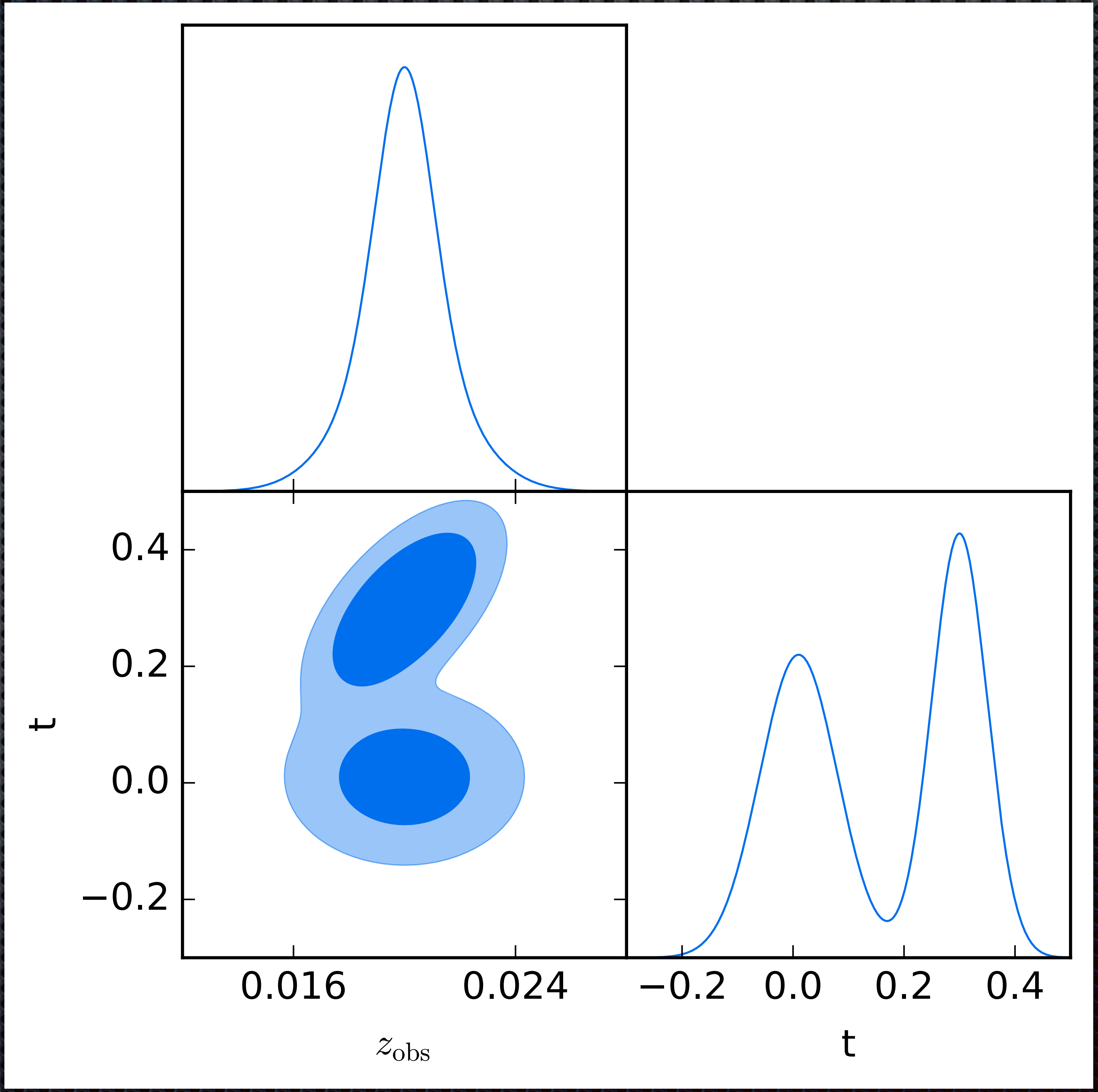
MCMC step

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10}A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8\Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8\Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023
f_{2000}^{143}	29.9 ± 2.9	30.4 ± 2.9	30.3 ± 2.8	29.5 ± 2.7	30.2 ± 2.7	30.0 ± 2.7
$f_{2000}^{143\times217}$	32.4 ± 2.1	32.8 ± 2.1	32.7 ± 2.0	32.2 ± 1.9	32.8 ± 1.9	32.6 ± 1.9
f_{2000}^{217}	106.0 ± 2.0	106.3 ± 2.0	106.2 ± 2.0	105.8 ± 1.9	106.2 ± 1.9	106.1 ± 1.8

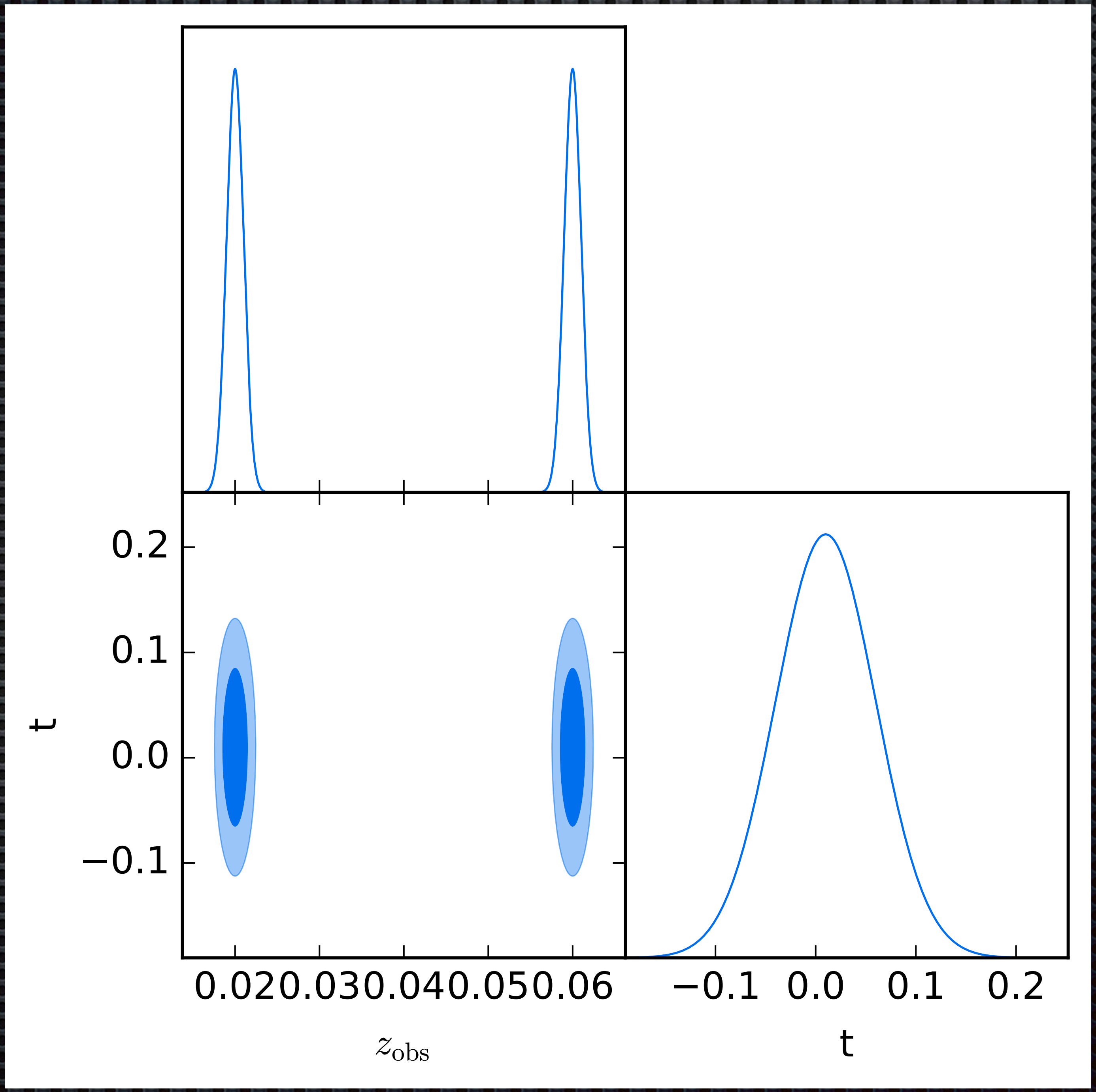
Gaussian



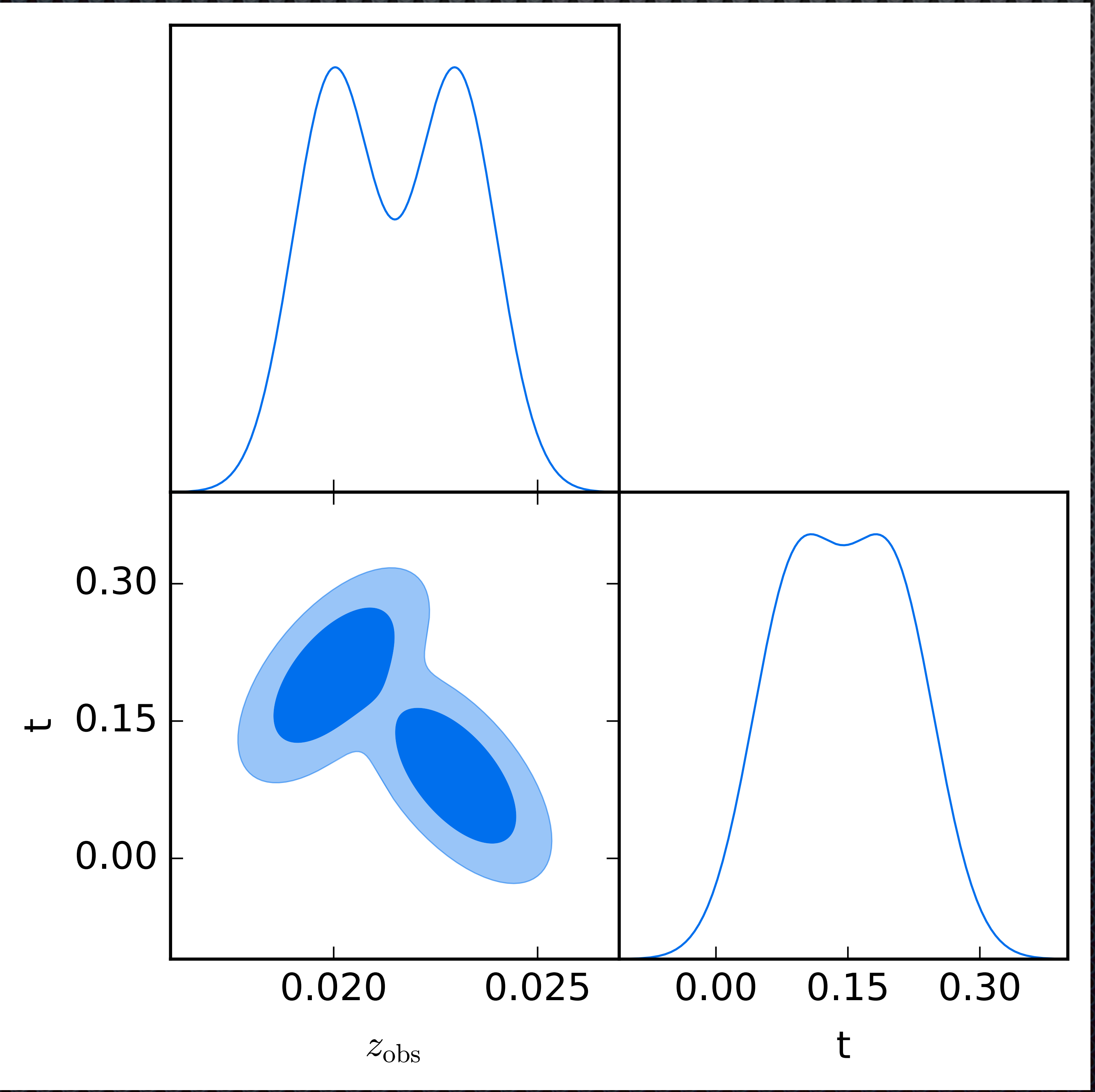
Bimodal



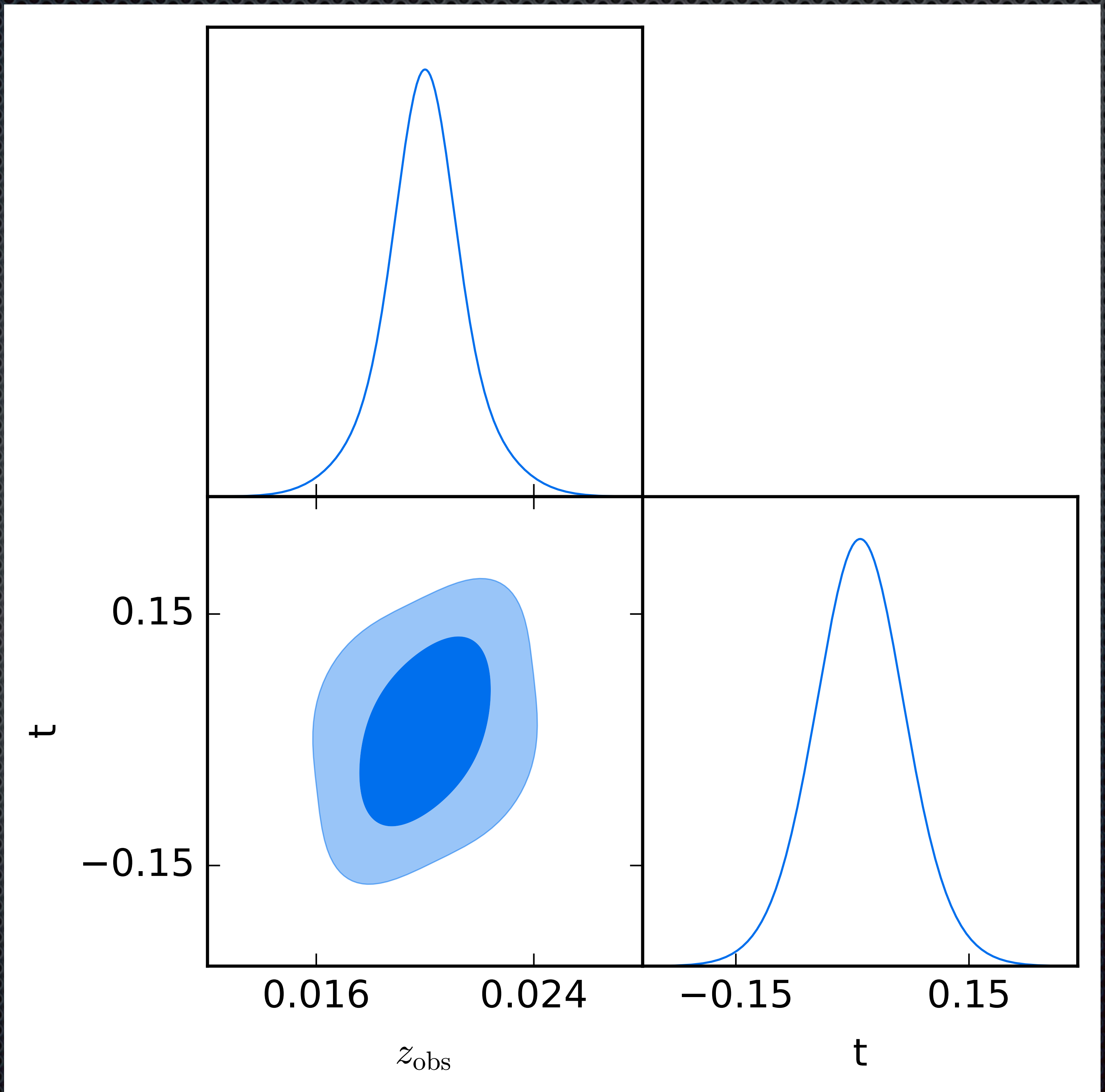
Bimodal



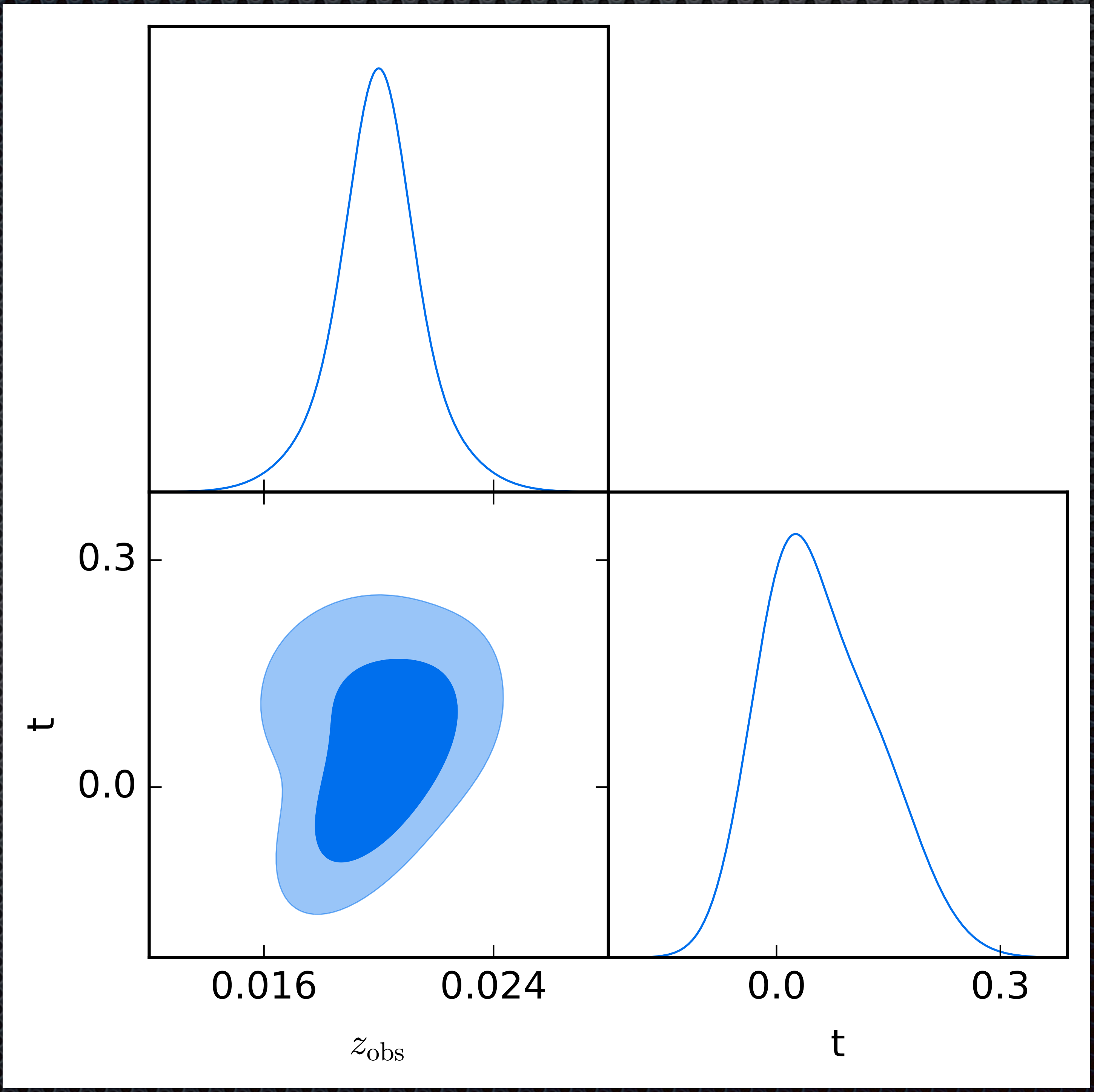
Hammer

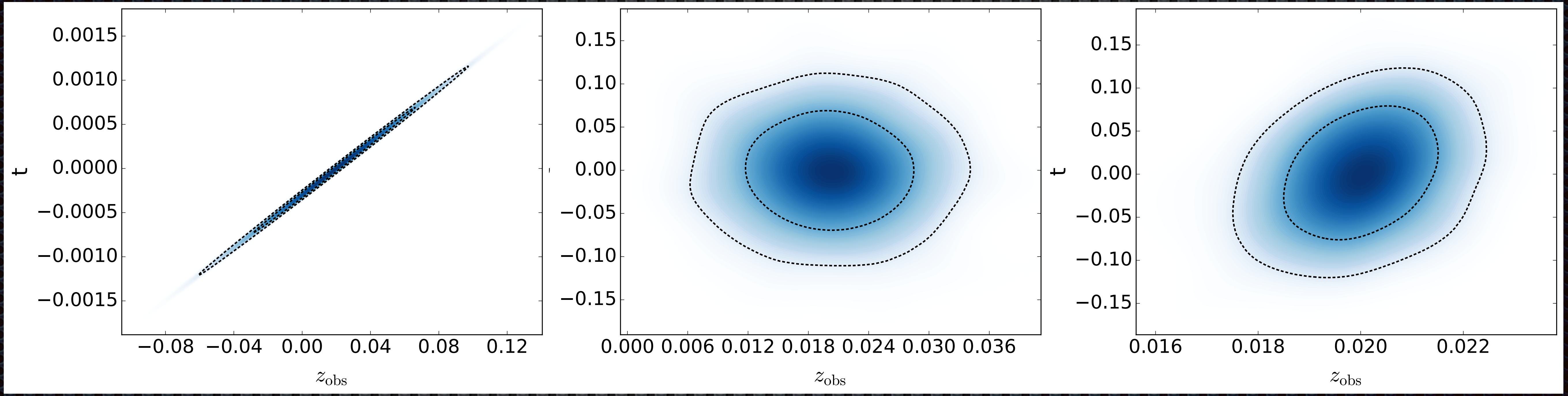


Broad tail



Skew




t
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−0.08 −0.04 0.00 0.04 0.08 0.12
z_{obs}
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.000 0.006 0.012 0.018 0.024 0.030 0.036
z_{obs}
t
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.016 0.018 0.020 0.022
z_{obs}

C=1 C=0 C=0.4

Other codes

- A strong competitor is **MontePython**, a python code that interfaces the Boltzmann solver **CLASS**. The advantage of CLASS over CAMB is that it matches the notation in Ma & Bertschinger, so it is easier to generalize
- A rising code is **cosmosis**, another python code that is extremely modular, which makes it easy to switch between MCMC samplers, Boltzmann codes
- Other samplers have been used in the past (CosmoPMC, PICO, etc.) but they do not seem to be in active development or use approximated methods
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