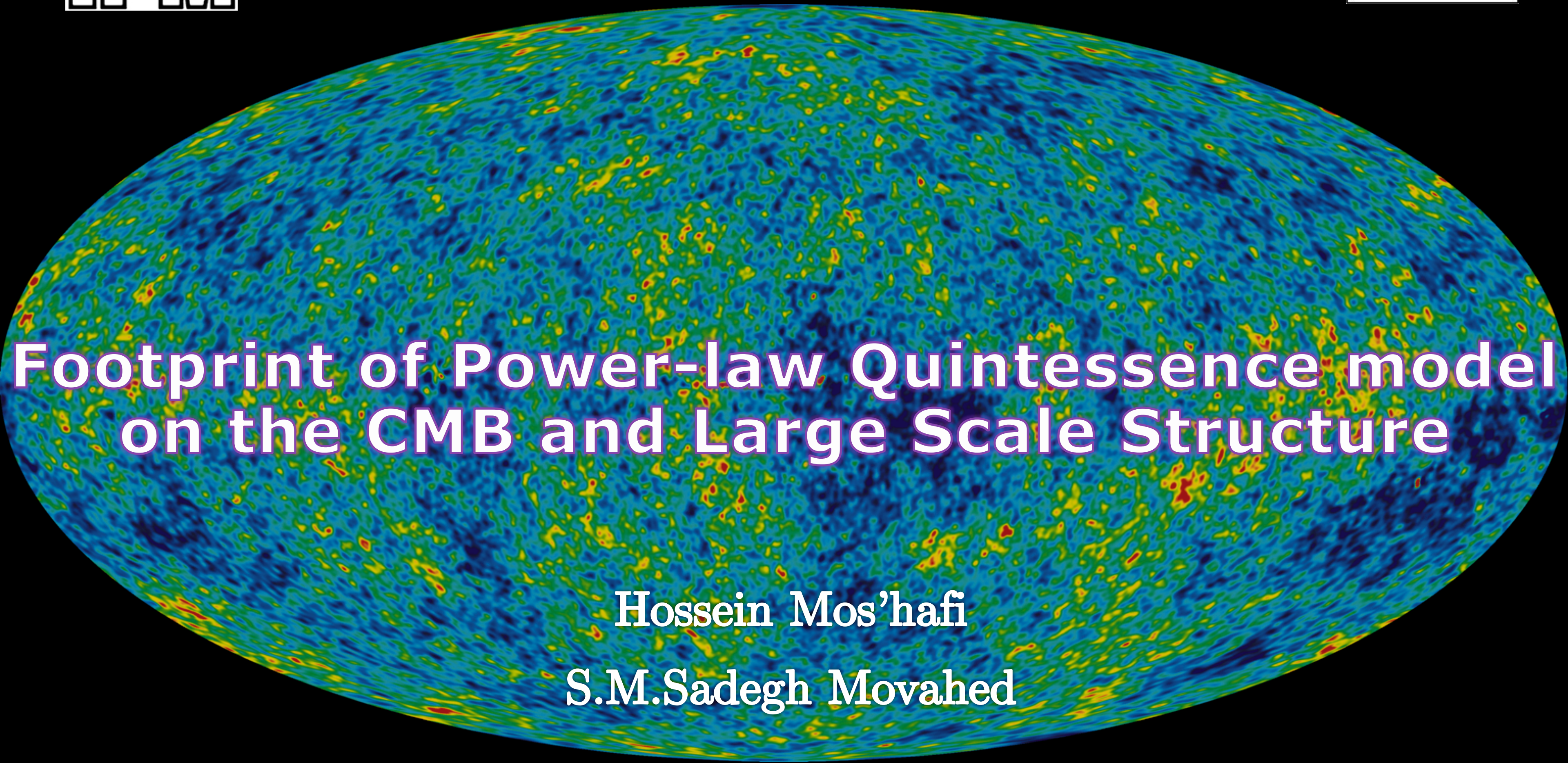
In the name of Allah



Footprint of Power-law Quintessence model on the CMB and Large Scale Structure

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Outline:

- Introduction to CMB
  - Observables
  - Anisotropies
  - Power Spectrums: Temperature, Primordial, Matter
- Variable Dark Energy models
  - Quintessence model
  - Power-law Quintessence model
- Observational Constraints
- Summary and Conclusions

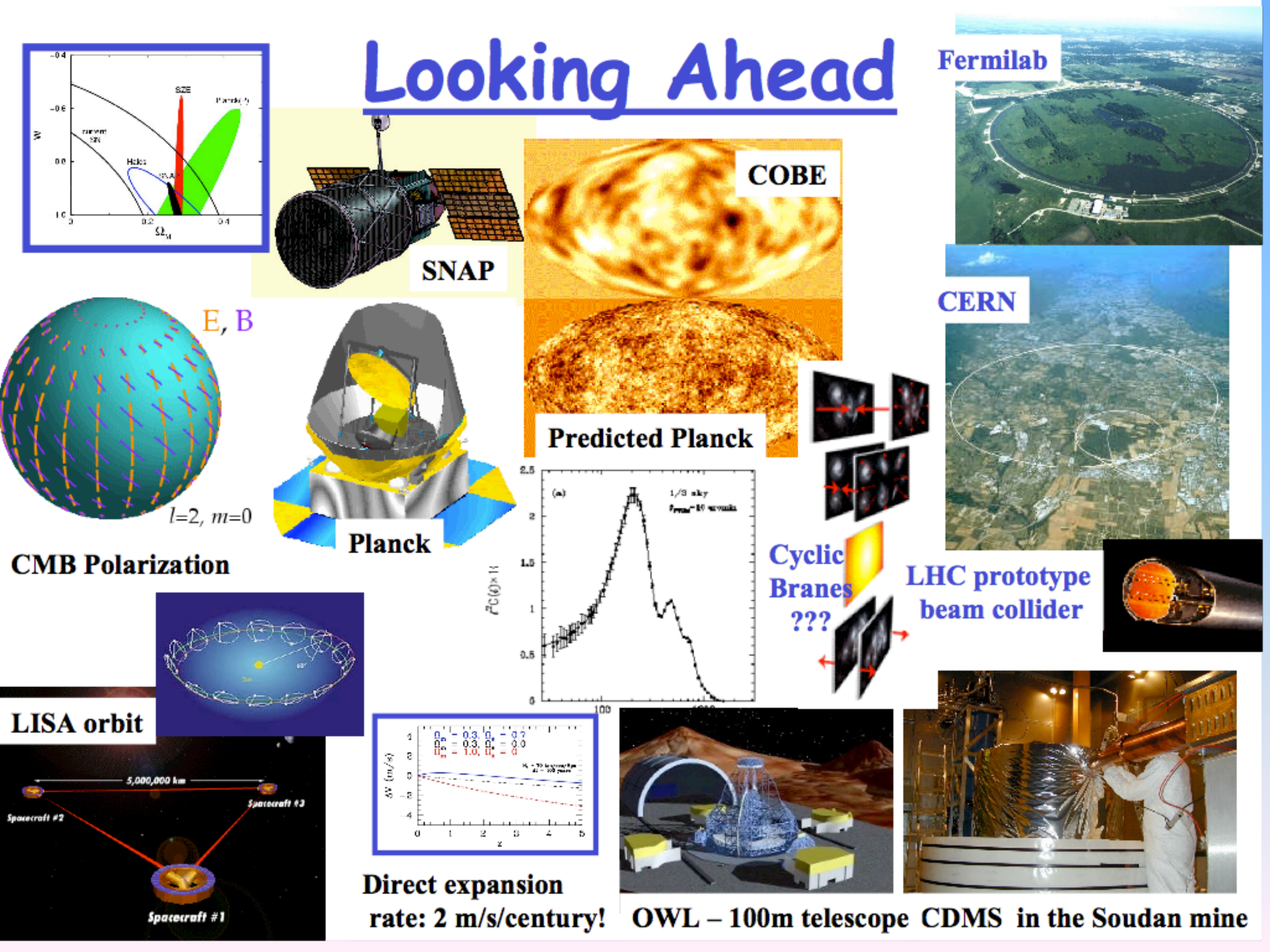
Observable parameters

- First set: Related to the background evolution. About 10 parameters:

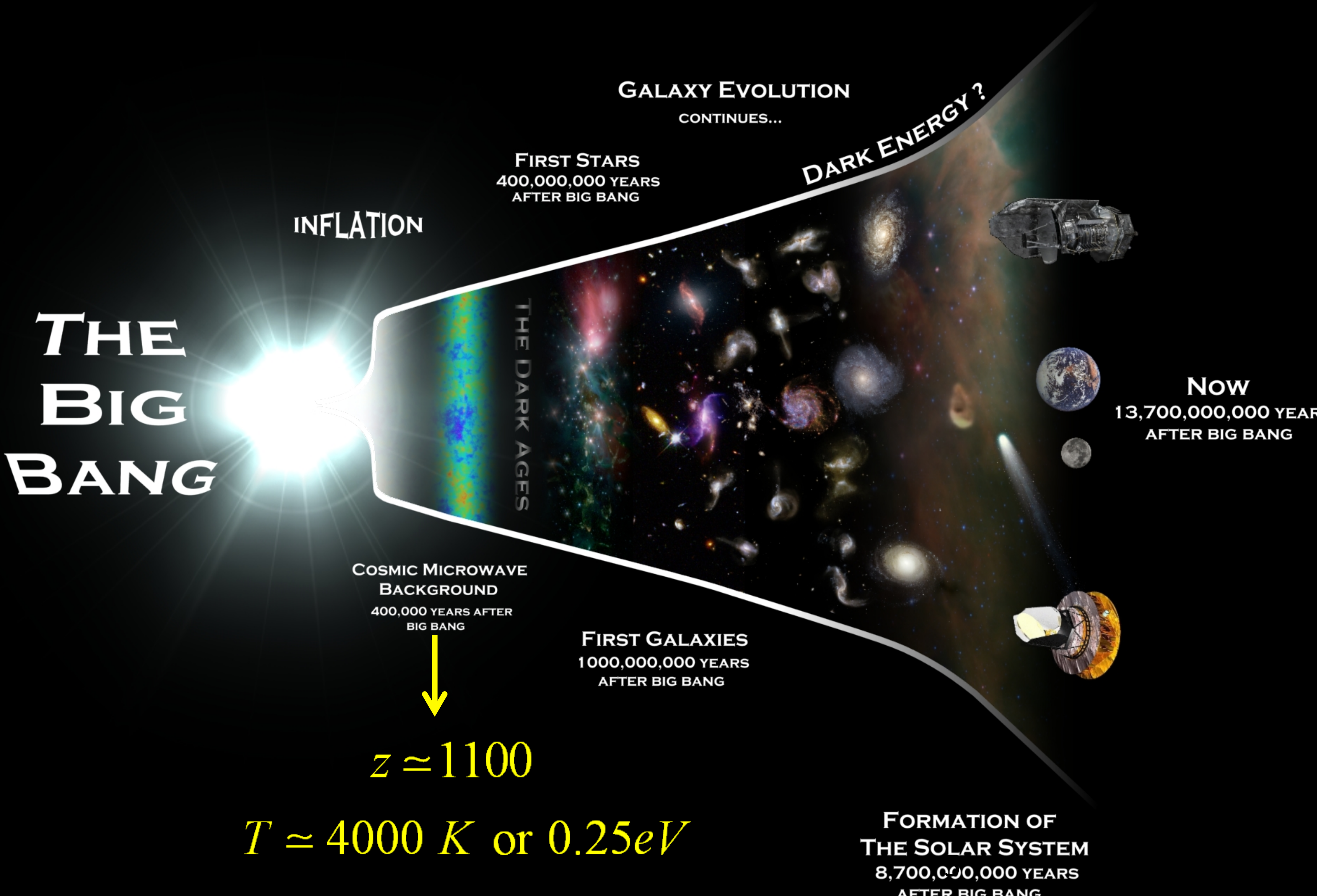
$$\Omega_{cdm}, \Omega_b, \Omega_\nu, \Omega_K, \Omega_\Lambda, w, t_0, H_0, q_0, T_{CMB}$$

- Second set: Describe deviation from perfect homogeneity and isotropy. About 6 parameters:

$$\sigma_8, A_s, A_t, n_s, n_t, dn / d\ln k$$


Looking Ahead
Fermilab
SNAP
COBE
CERN
E, B
l=2, m=0
Planck
Predicted Planck
CMB Polarization
Cyclic Branes ???
LHC prototype beam collider
LISA orbit
5,000,000 km
Spacecraft #2
Spacecraft #3
Spacecraft #1
Direct expansion rate: 2 m/s/century!
OWL – 100m telescope
CDMS in the Soudan mine

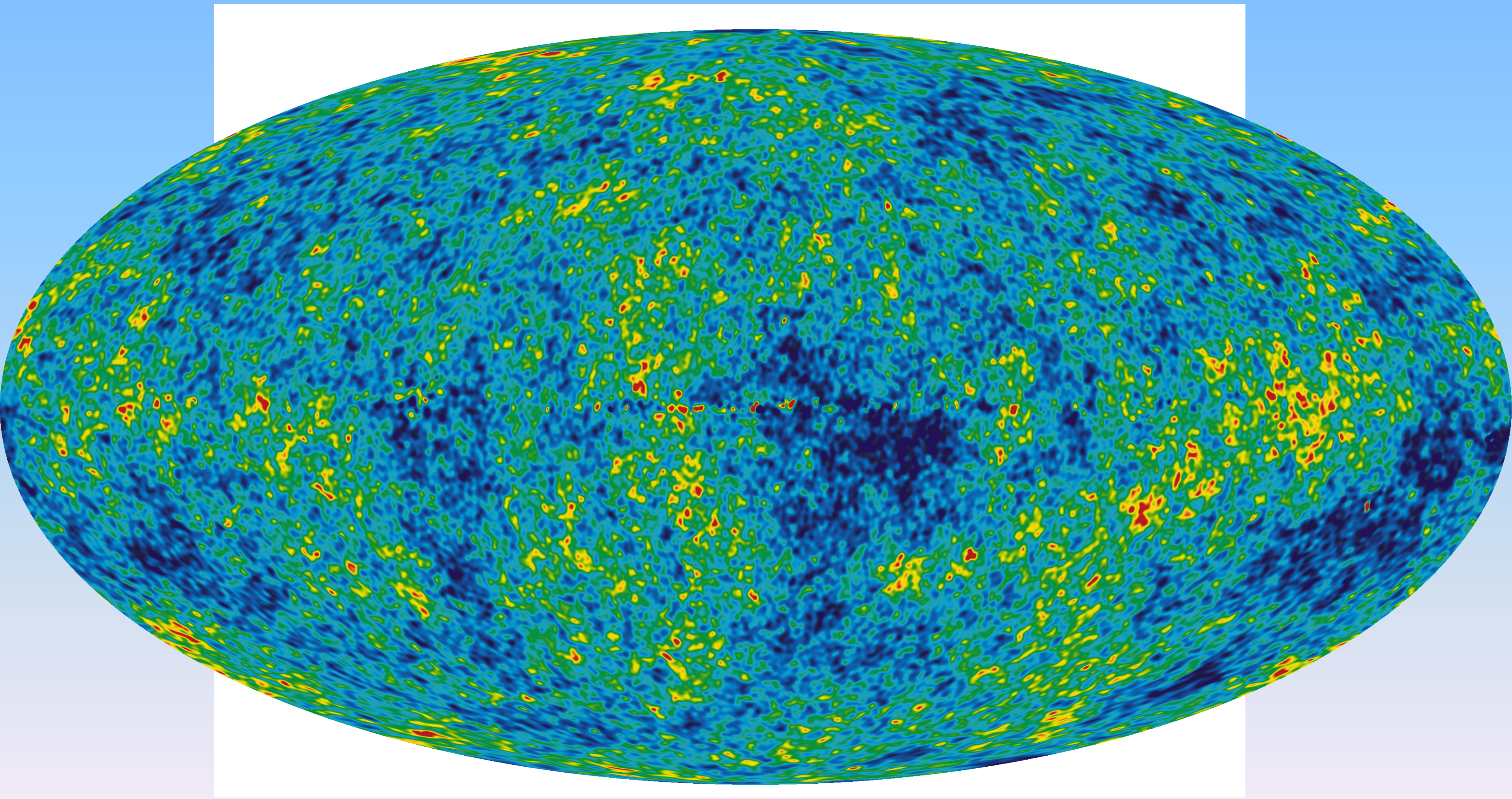
CMB Physics



$$z \simeq 1100$$

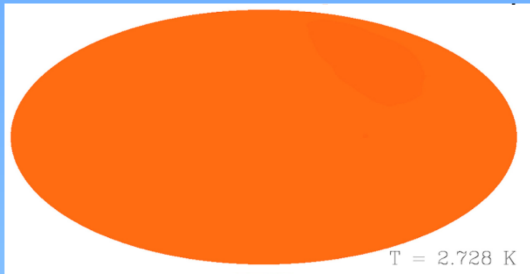
$$T \simeq 4000\ K \text{ or } 0.25 eV$$

Observed T as a random field on the sphere

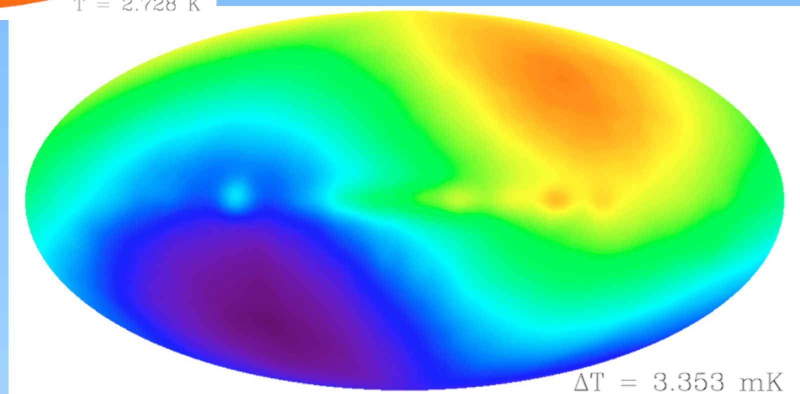


CMB Anisotropies

- **Primary anisotropies**
  - Sachs-Wolfe effect
  - Doppler effect
  - Intrinsic temperature variations
  - Cosmic Strings
- **Secondary anisotropies**
  - Integrated Sachs-Wolfe (ISW) effect
  - Sunyaev- Zel' dovich (SZ) effect
  - Gravitational Lensing
  - Cosmic Strings (Gravitational Waves)



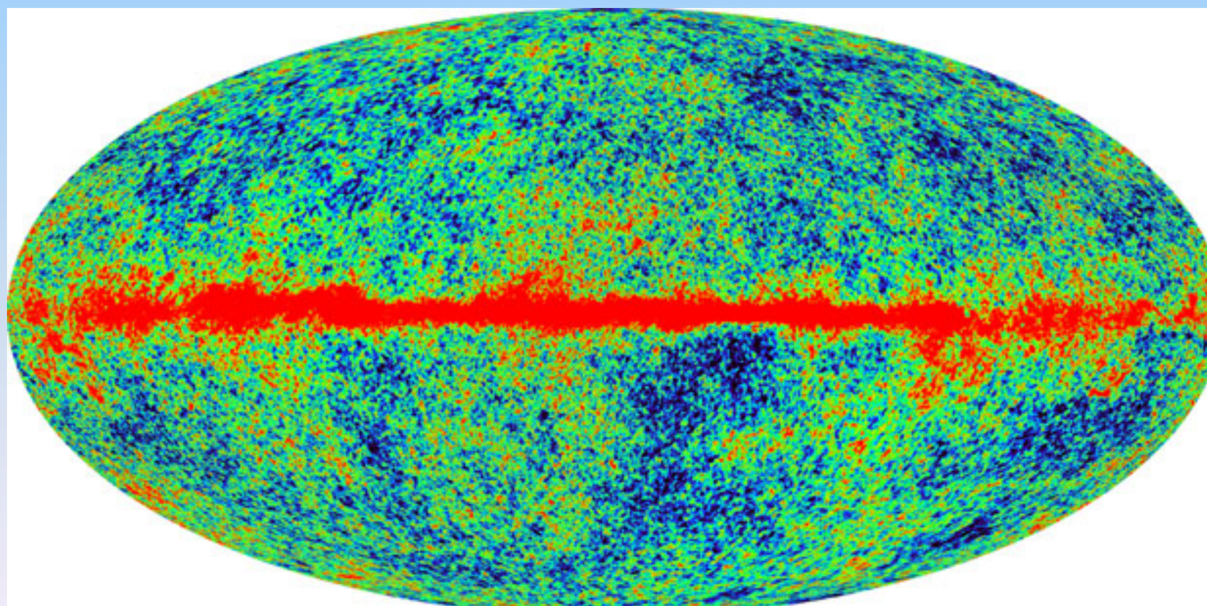
(almost) uniform 2.726K blackbody



Dipole (local motion)

O(10^{-5}) perturbations (structures)

Observations: the microwave sky today



Source: NASA/WMAP Science Team

Calculation of theoretical perturbation evolution

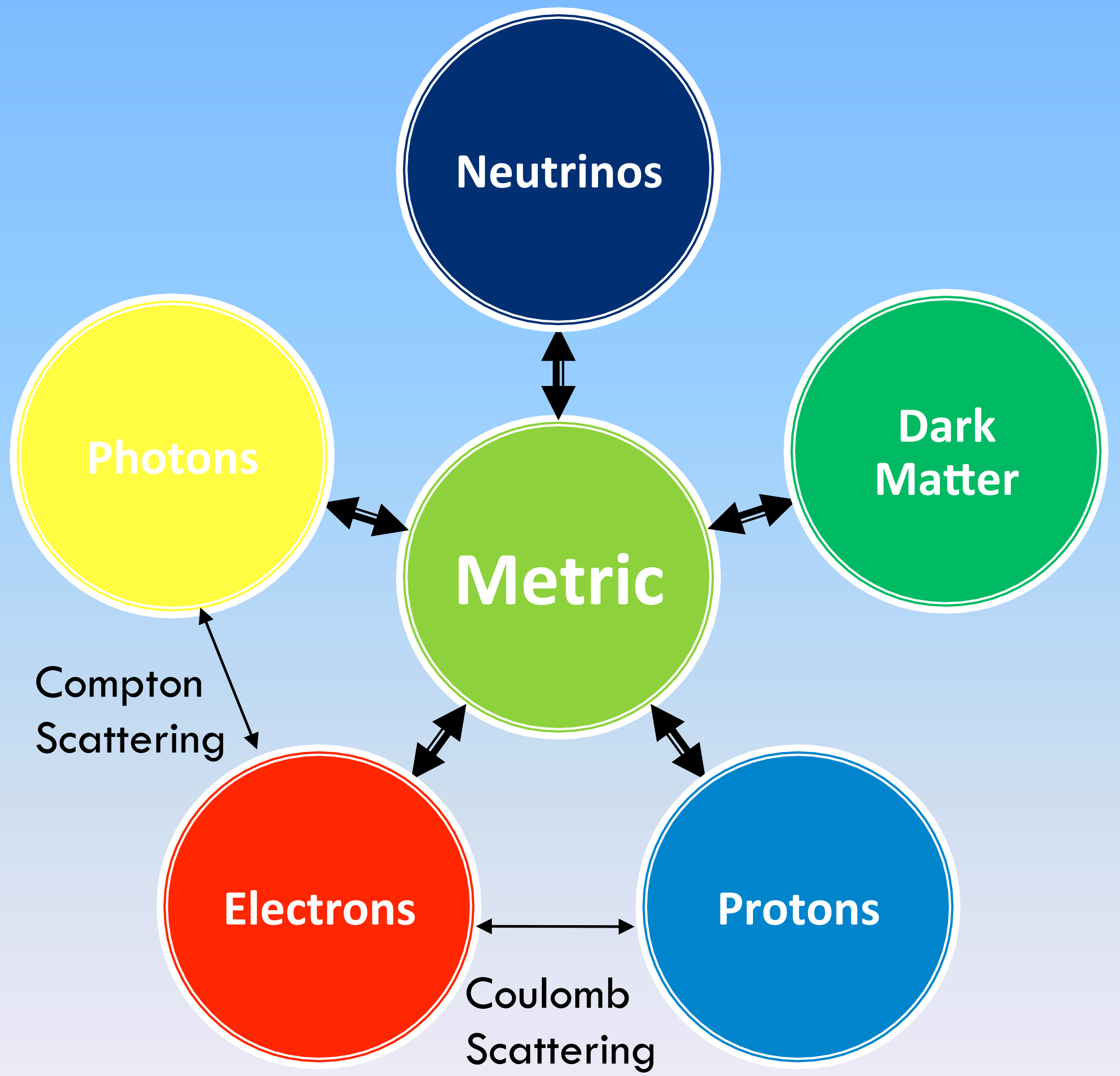
Perturbations $O(10^{-5})$

- Simple linearized equations are very accurate (except for small scales)
- We can use real or Fourier space
- Fourier modes evolve independently: simple to calculate accurately

Physics Ingredients:

- ❑Thomson scattering (non-relativistic electron-photon scattering)
  - tightly coupled before recombination: 'tight-coupling' approximation (baryons follow electrons because of very strong e-m coupling)
- ❑Background recombination physics (Saha/full multi-level calculation)
- ❑Linearized General Relativity
- ❑Boltzmann equation (how angular distribution function evolves with scattering)

Sources of perturbations



Einstein- Boltzmann equations

Boltzmann equation

$$\frac{df}{dt} = C[f]$$

Photons (2 equations)

$$1)\ \dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi]$$

$$2)\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}[-\Theta_P + \frac{1}{2}(1 - \mathcal{P}_2(\mu))\Pi]$$

$$\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

CDM (2 equations)

$$3)\ \dot{\delta} + ikv = -3\dot{\Phi}$$

$$4)\ \dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$$

Baryons (2 equations)

$$5)\ \dot{\delta}_b + ikv_b = -3\dot{\Phi}$$

$$6)\ \dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]$$

$$\frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$$

Neutrinos (1 equation)

$$7)\ \dot{\mathcal{N}} + ik\mu\mathcal{N} = -\dot{\Phi} - ik\mu\Psi$$

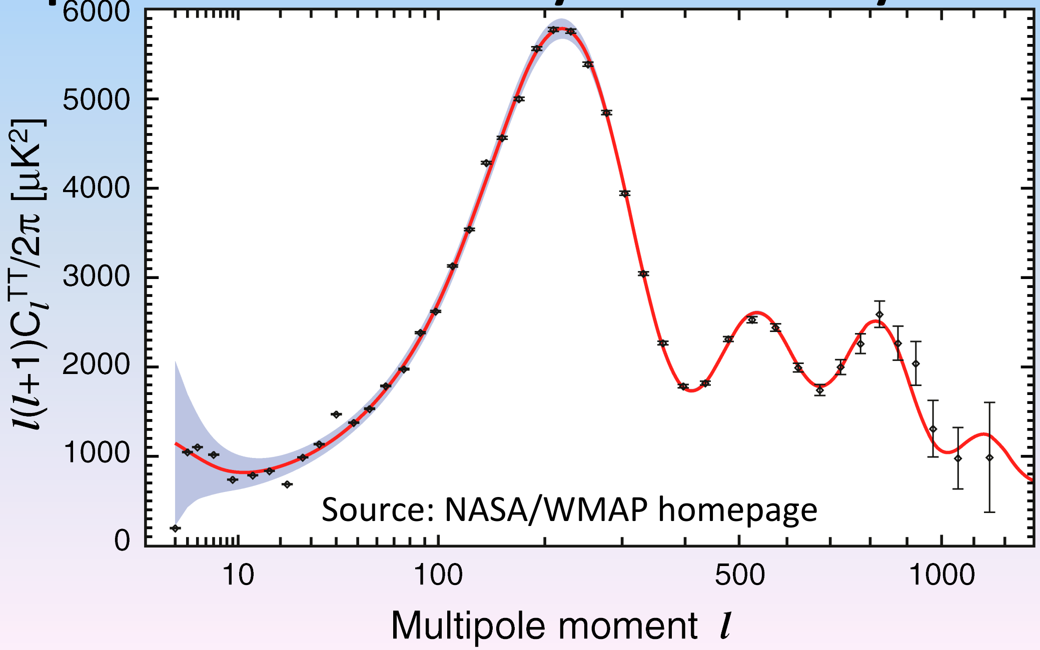
Einstein-perturbed equation (2 equation)

$$8)\ k^2\Phi + 3\frac{\dot{a}}{a}(\dot{\Phi} - \Psi\frac{\dot{a}}{a}) = 4\pi G a^2[\rho_{CDM}\delta + \rho_b\delta_b + 4(\rho_\gamma\Theta_0 + \rho_\nu\mathcal{N}_0)]$$

$$9)\ k^2(\Phi + \Psi) = -32\pi G a^2(\rho_\gamma\Theta_2 + \rho_\nu\mathcal{N}_2)$$

Power Spectrum Calculation by using Computer codes

- CMBFAST, CAMB, CMBEASY,…
- CAMB , Open source code : http://camb.info
- CAMB codes Should be modified in right way to solve equations for any arbitrary model



Why Power Spectrum?

- Temperature fluctuation is a random field
- We are interested in finding dominant modes superimposed to make final map
- Fourier or Legendre transformation of correlation function

$$C_x(i,j) = \langle x(i).x(j) \rangle$$
$$= C_x(|\, i - j \,|) \equiv C_x(\tau) = \langle x(i).x(i+\tau) \rangle$$

$$S_x(\omega) = \frac{1}{2T} \int_{-T}^{T} C_x(\tau) e^{i\omega x} d\tau$$

$$C_x(\tau) = \int_{-T}^{T} S_x(\omega) e^{-i\omega x} d\omega$$

Primordial power spectrum

Primordial Power Spectrum for Scalar Perturbations :

$$P_{\Phi}(k) = \frac{8\pi}{9k^3} \frac{H^2}{\epsilon m_{pl}^2}\bigg|_{aH=k} \equiv \frac{50\pi^2}{9k^3}\left(\frac{k}{H_0}\right)^{n-1} \delta_H^2 \left(\frac{\Omega_m}{D_1(a=1)}\right)^2$$

Primordial Power Spectrum for Tensor Perturbations :

$$P_h(k) = \frac{8\pi}{k^3} \frac{H^2}{m_{pl}^2}\bigg|_{aH=k} \equiv A_T k^{n_T - 3}$$

The Power Spectrum

$$\langle \Theta(\vec{k},\hat{p})\Theta(\vec{k}',\hat{p}')\rangle = \langle \delta(\vec{k})\delta^*(\vec{k}')\rangle \frac{\Theta(\vec{k},\hat{p})}{\delta(\vec{k})}\frac{\Theta^*(\vec{k}',\hat{p})}{\delta(\vec{k}')}$$

$$= (2\pi)^3\delta^3(\vec{k}-\vec{k}')P(k)\frac{\Theta(k,\hat{k}.\hat{p})}{\delta(k)}\frac{\Theta^*(k,\hat{k}.\hat{p}')}{\delta^*(k)}$$

$P(k)$ → primordial power spectrum

$$\left\langle a_{lm}a^*_{l'm'}\right\rangle = \delta_{ll'}\delta_{mm'}C_l$$

$$C_l = \int \frac{d^3k}{(2\pi)^3}P(k)\int d\Omega Y^*_{lm}(\hat{p})\frac{\Theta(k,\hat{k}.\hat{p})}{\delta(k)}\int d\Omega' Y^*_{lm}(\hat{p}')\frac{\Theta(k,\hat{k}.\hat{p}')}{\delta^*(k)}$$

$$C_l = \frac{2}{\pi}\int_0^\infty dk k^2 P(k)\left|\frac{\Theta_l(k)}{\delta(k)}\right|^2$$

The Power Spectrum

$$C_l = \int \frac{d^3k}{(2\pi)^3} P(k)\Theta_l^2(k)$$

With a Harrison- Zel' dovich spectrum predicted by inflation, P(k) is:

$$\frac{k^3}{2\pi^2} P(k) = \left(\frac{k}{H_0}\right)^{n_s - 1}$$

$$C_l = \int_0^\infty \left(\frac{k}{H_0}\right)^{n_s - 1} \Theta_l^2(k) \frac{dk}{k}$$

n_s → Spectral Index

n_s is close to 1 from inflation

Matter power spectrum

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x})}{\bar{\rho}}$$

Density contrast

$$\delta(\vec{x}) = \frac{1}{(2\pi)^3}\int d\vec{k}^3 e^{i\vec{k}.\vec{x}}\delta\vec{k}$$

$$\langle\delta(\vec{x})\delta(\vec{x}')\rangle = \frac{1}{(2\pi)^6}\int d\vec{k}^3\int d\vec{k}'^3 e^{i\vec{k}.\vec{x}}e^{i\vec{k}'.\vec{x}}\langle\delta(\vec{k})\delta(\vec{k}')\rangle$$

$$\langle\delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')P(\vec{k})$$

$T(\vec{k})$
Transfer function: that determine the evolution of potential in the radiation-matter equality epoch

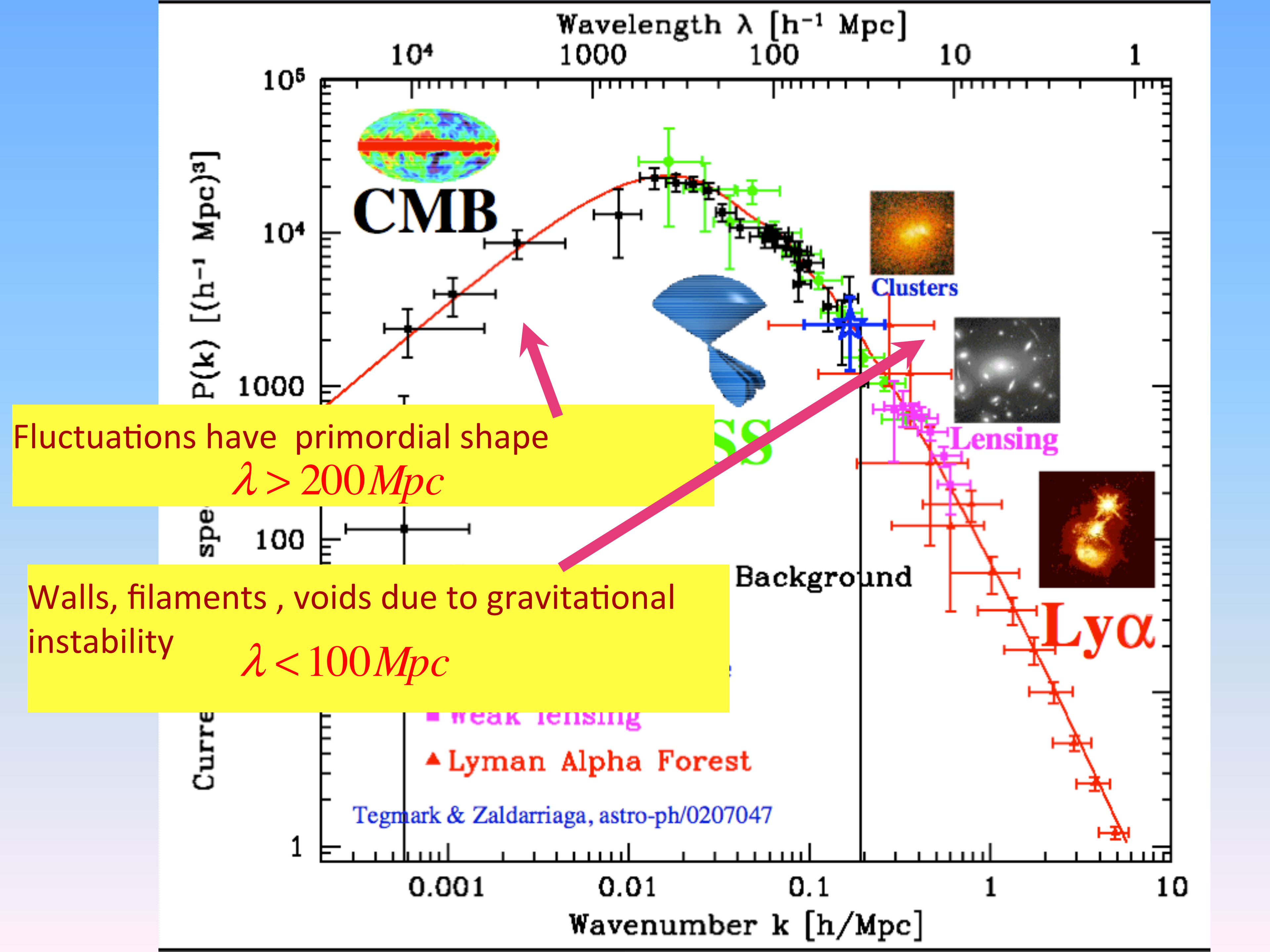
$$P(\vec{k},a) = 2\pi^2\delta_H^2 T^2(\vec{k})\frac{k^n}{H_0^{3+n}}\left[\frac{D(a)}{D(a=1)}\right]^2$$

Matter Power Spectrum at late times

$$\frac{d^2D(a)}{da^2} + \frac{dD(a)}{da}\left[\frac{\ddot{a}}{a^2} + \frac{2H(a;l)}{\dot{a}}\right] - \frac{3H_0^2}{2\dot{a}^2a^3}\Omega_m D(a) = 0$$

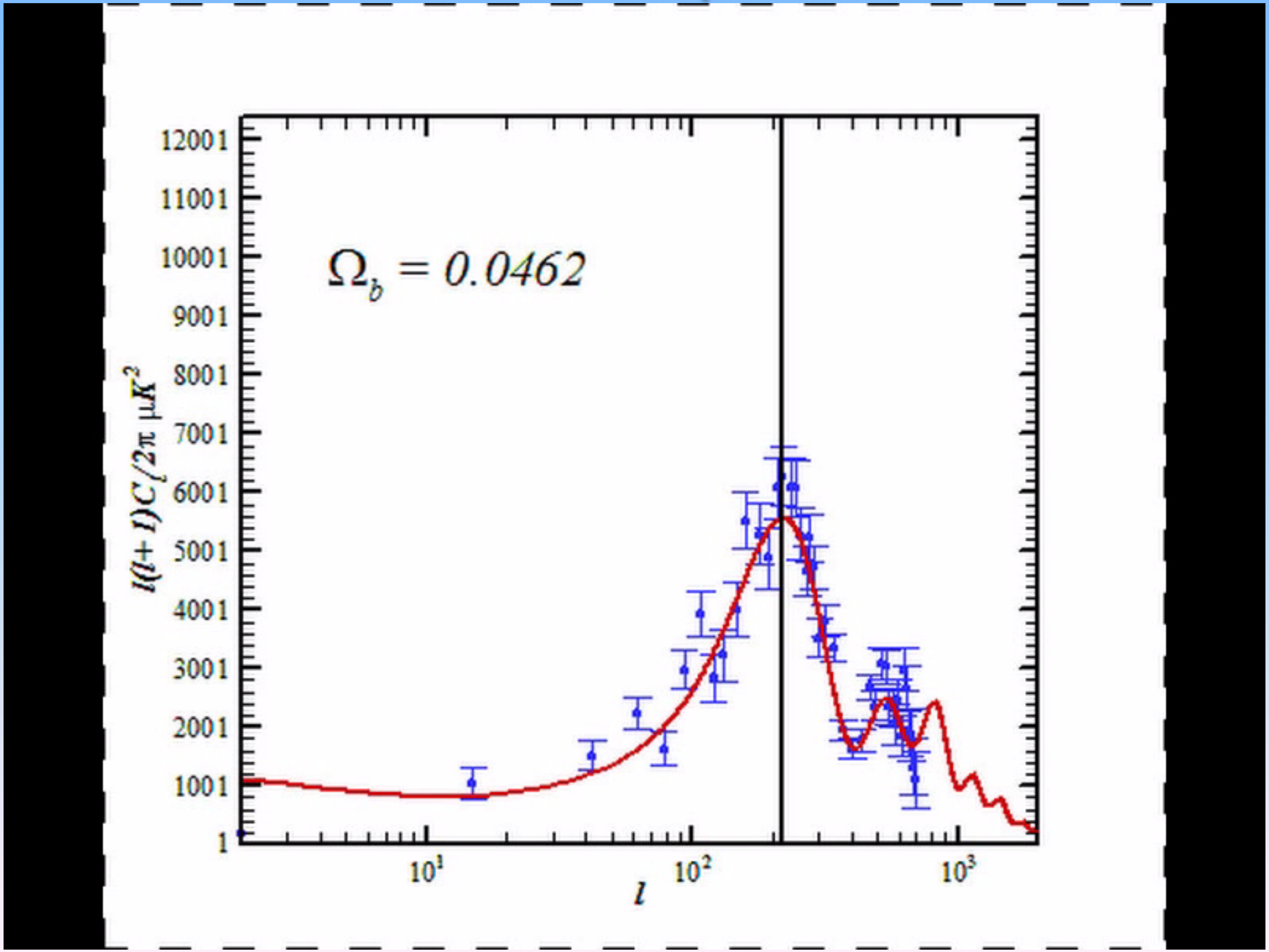
$D(a)$:Growth factor

δ_H : Scalar amplitude at horizon crossing

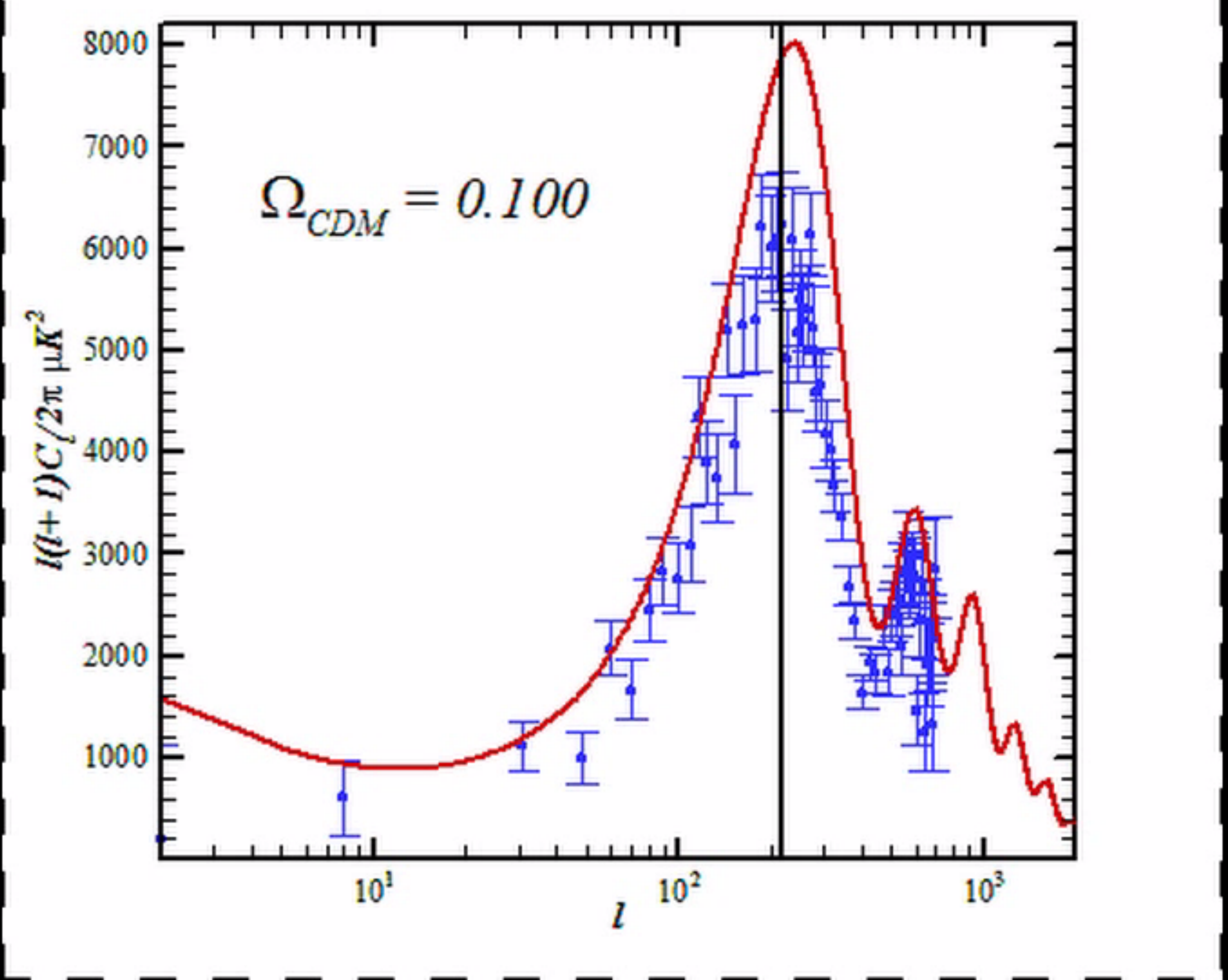


Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
1
P(k) [(h⁻¹ Mpc)³]
CMB
Clusters
Lensing
Lyα
SS
Background
Lyman Alpha Forest
Tegmark & Zaldarriaga, astro-ph/0207047
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]
Fluctuations have primordial shape
λ > 200 Mpc
Walls, filaments , voids due to gravitational instability
λ < 100 Mpc

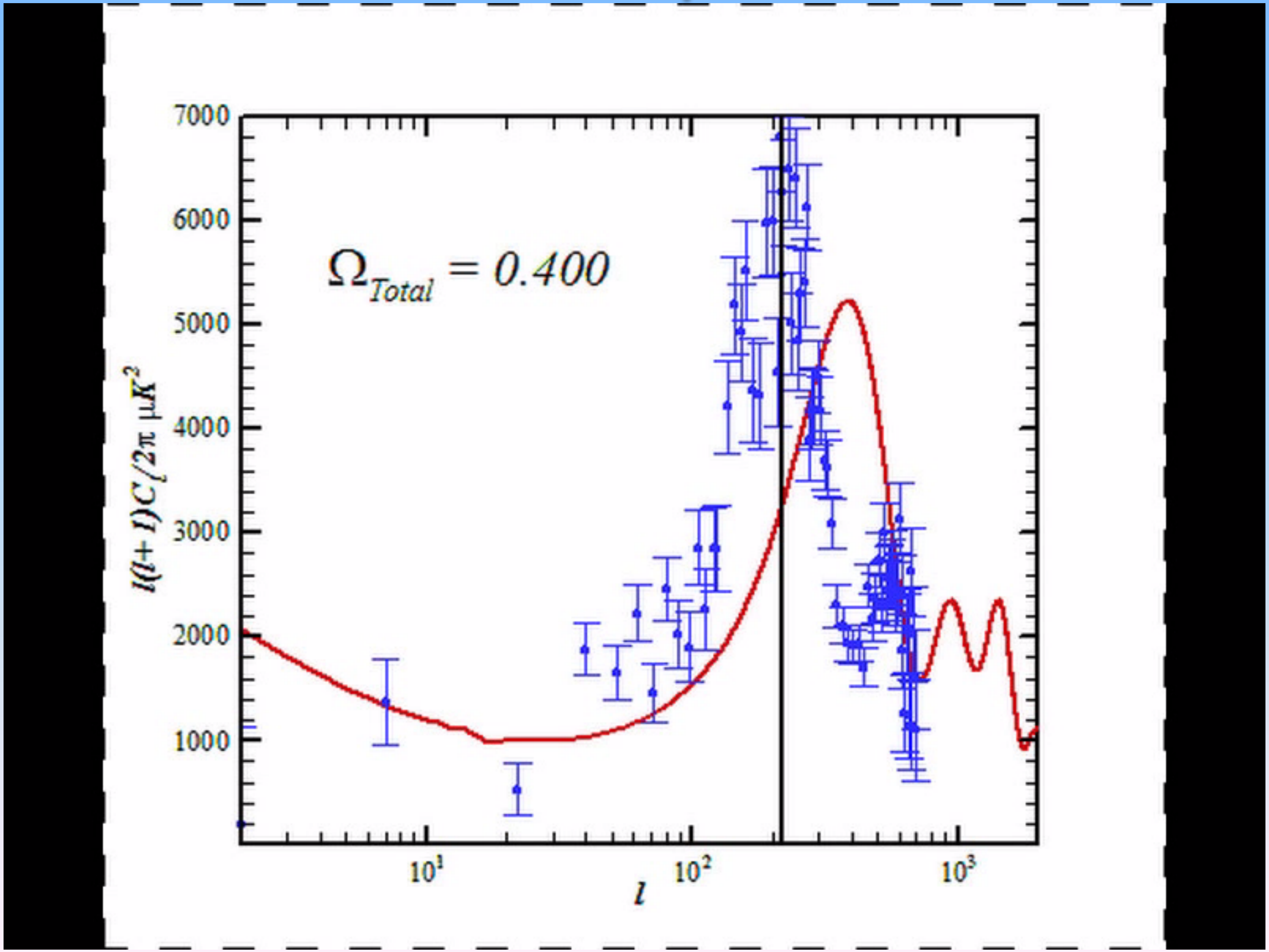
Effect of Baryon density on the Power Spectrum



Effect of CDM density on the Power Spectrum



Effect of Dark Energy density on the Power Spectrum



Variable Dark Energy models

Cosmic Coincidence:

Why are the energy densities of dark energy and dark matter nearly equal today?

Fine-tuning:

Why the observed value of dark energy 10^{-47} GeV is very different from vacuum energy 10^{71} GeV ?

Power-law Quintessence model

$$w(a) = w_0 a^\alpha (1 + \ln a^\alpha)$$

$$\text{Generalized EOS} \rightarrow \bar{w}(a;\alpha, w_0) = \frac{\int_1^a w(a';\alpha, w_0) d\ln(a')}{\int_1^a d\ln(a')}$$

$$\bar{w}(a;\alpha, w_0) = w_0 a^\alpha$$

$$\rho_\lambda(z;\alpha, w_0) = \rho_\lambda(1+z)^{3[1+\bar{w}(a)]}$$

$$H = H_0\sqrt{\Omega_r a^{-4} + (\Omega_{dm} + \Omega_b)a^{-3} + \Omega_Q a^{-3(1+\bar{w}(a))} + \Omega_K a^{-2}}$$

Quintessence model

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \Rightarrow \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = 0$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi}$$

$$\tilde{V} \equiv \frac{V}{\rho_\phi}$$

$$\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(1 + w_\phi)\rho_\phi$$

$$V(\phi) = \frac{1}{2}(1 - w_\phi)\rho_\phi$$

$$\tilde{\phi} \equiv \frac{\phi}{M_{Pl}}$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

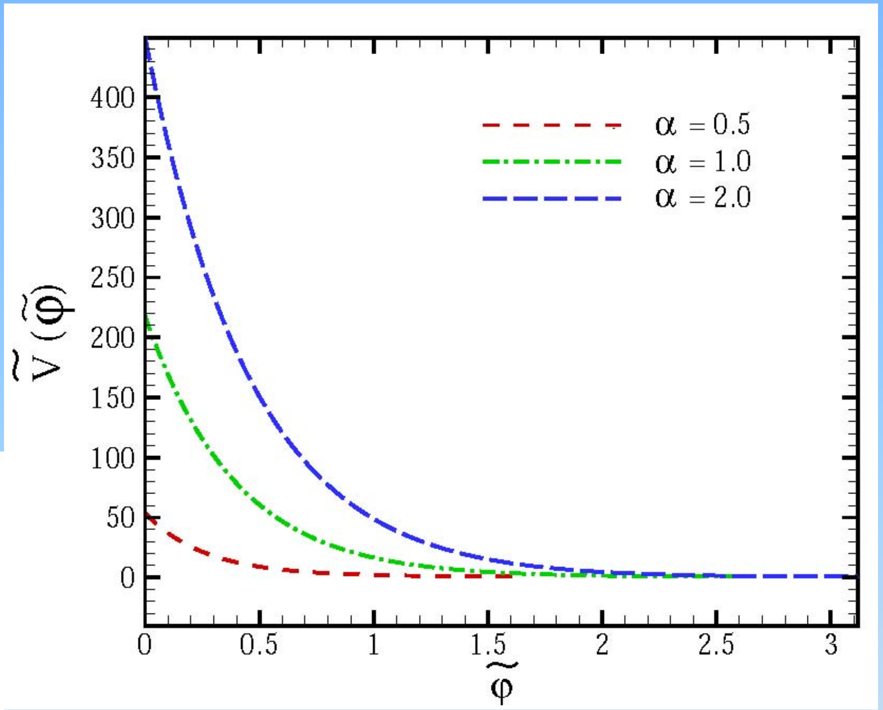
$$H^2(z;\{l\}) = H_0^2[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\phi U(z;\{l\})]$$

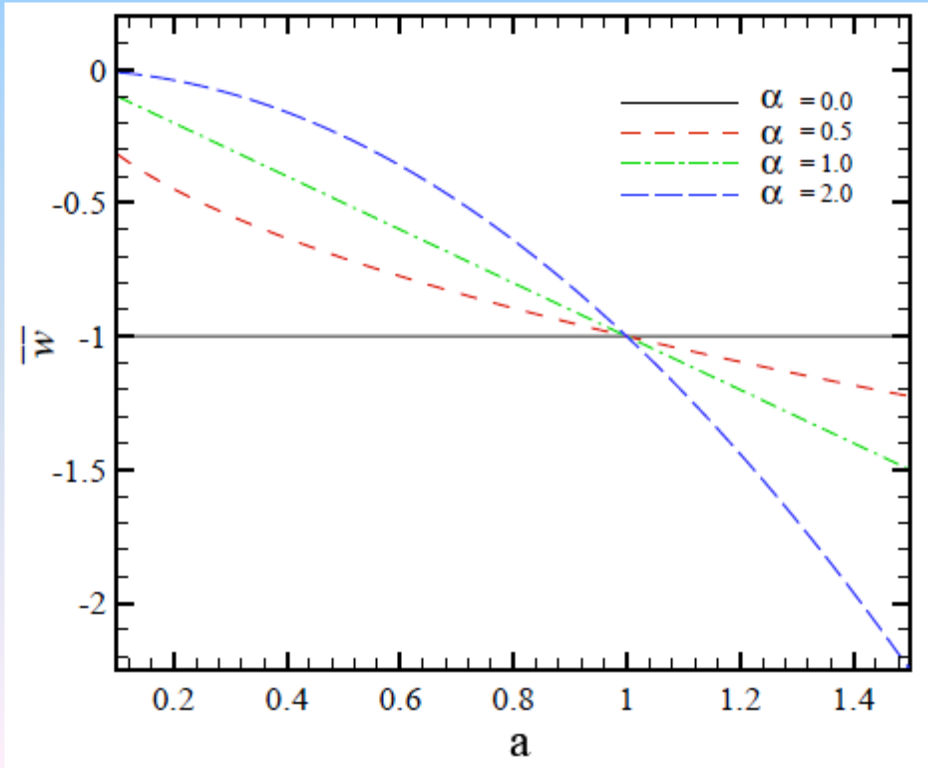
$$\rho_\phi(z) = \rho_\phi^0 e^{3\int_0^z (1+w_\phi(z;\{l\}))d\ln(1+z)} \equiv \rho_\phi^0 U(z;\{l\})$$

$$V(\phi(z)) = \frac{1}{2}(1 - w_\phi(z;\{l\}))\rho_\phi^0 U(z;\{l\})$$

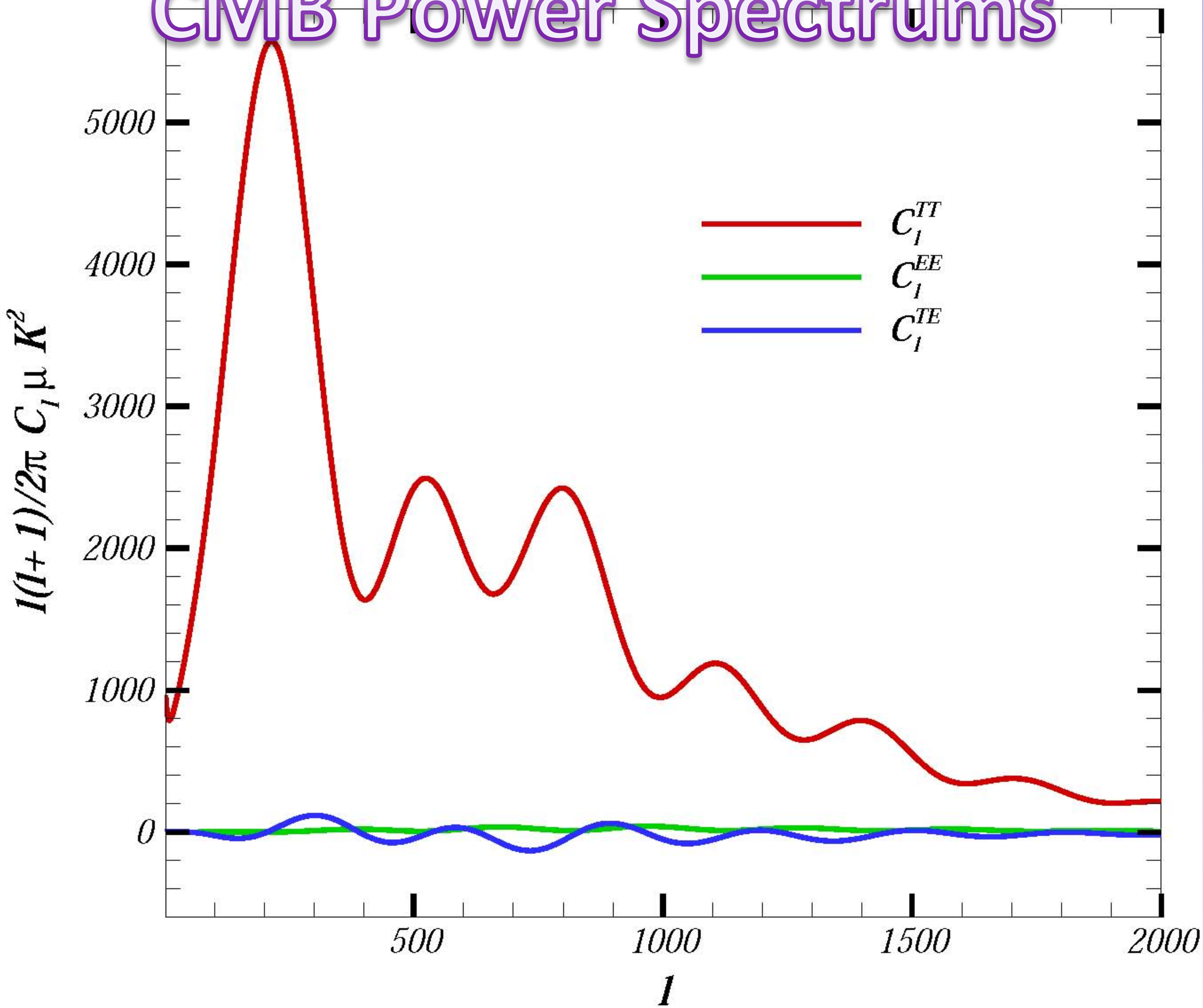
$$\frac{d\phi}{dz} = \pm\frac{(1 + w_\phi(z;\{l\}))^{1/2}}{(1+z)H(z;\{l\})}\rho_\phi^{1/2}$$

Power-law Quintessence model

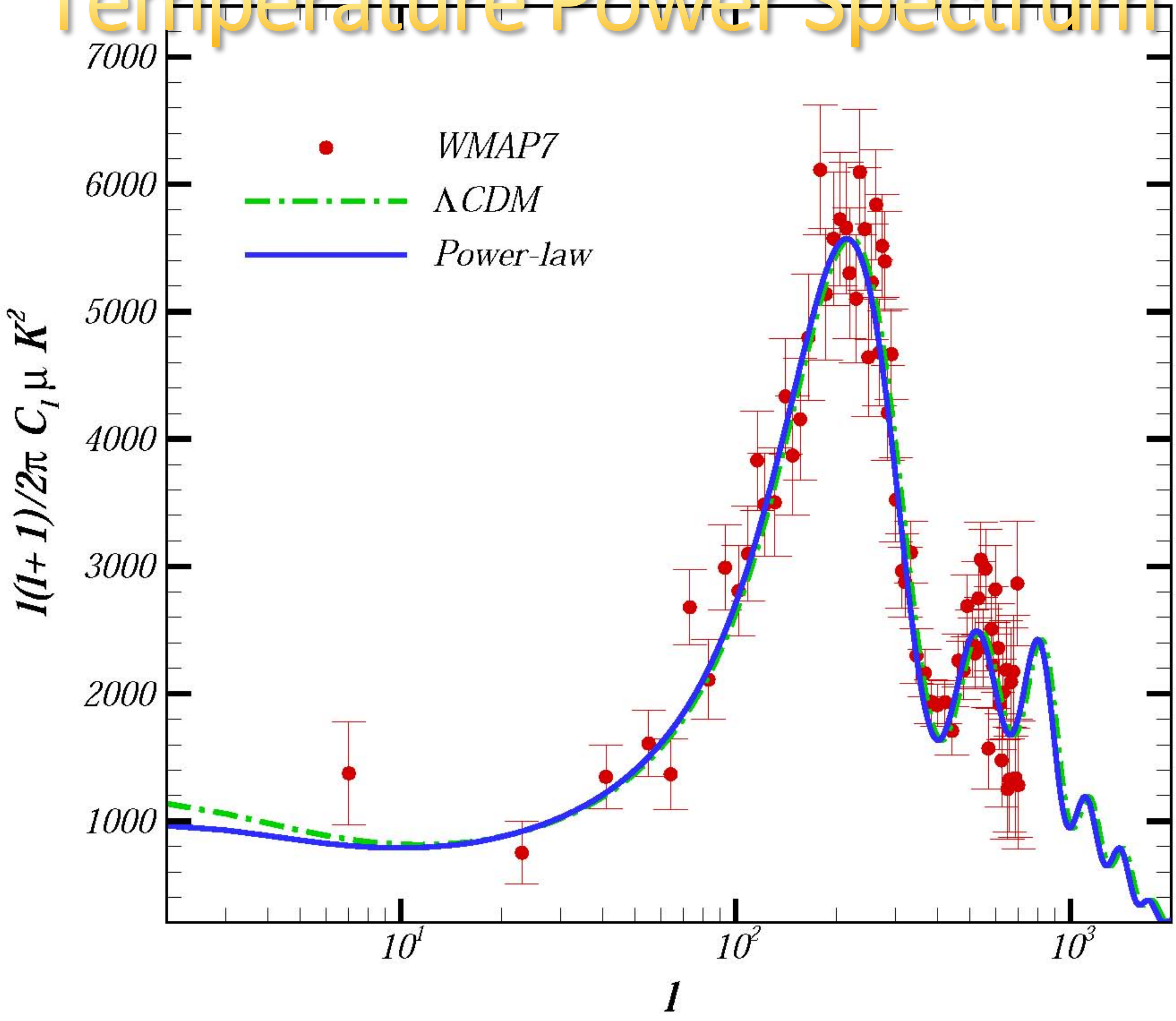




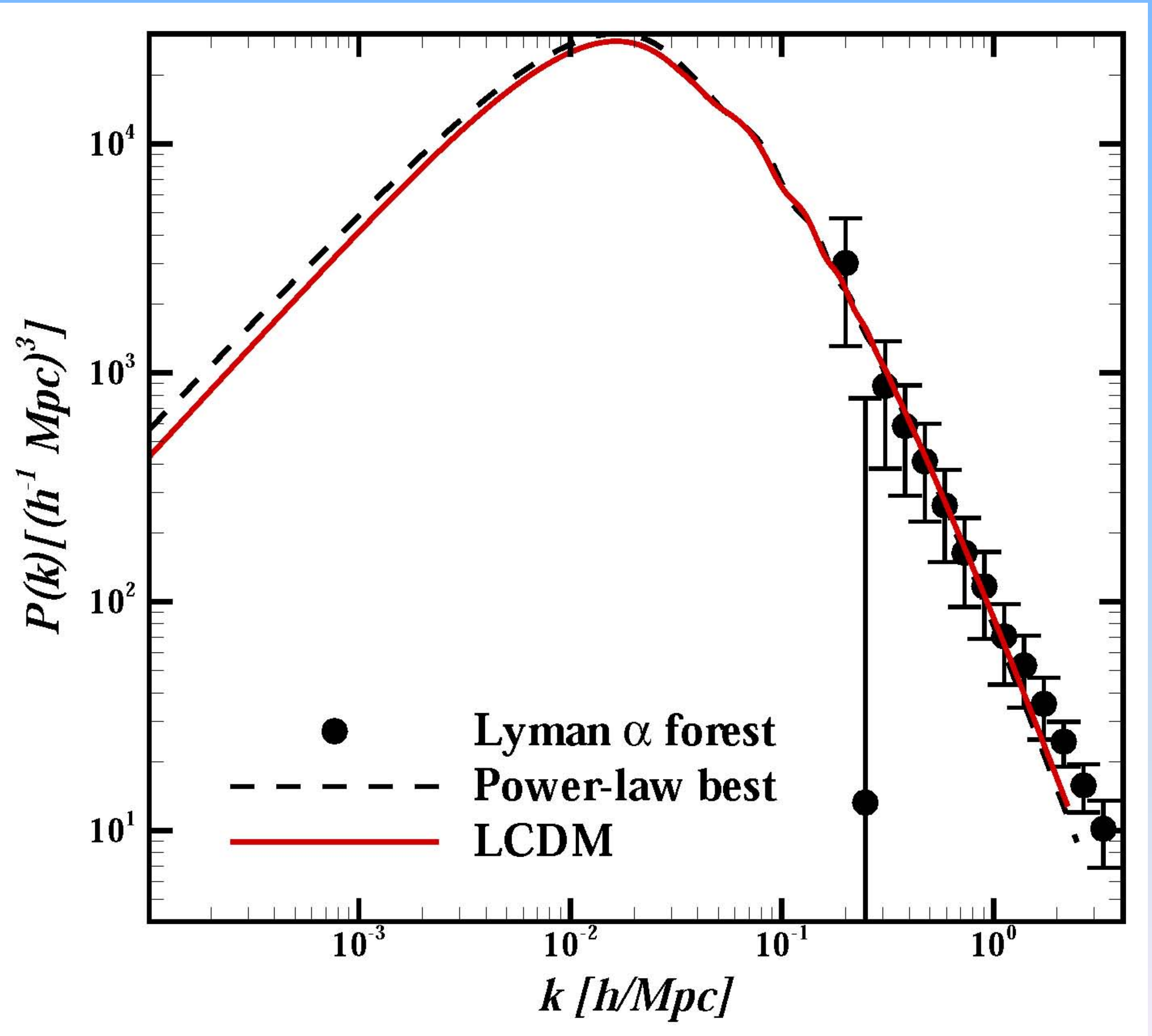
CMB Power Spectrums



Temperature Power Spectrum



Matter Power Spectrum

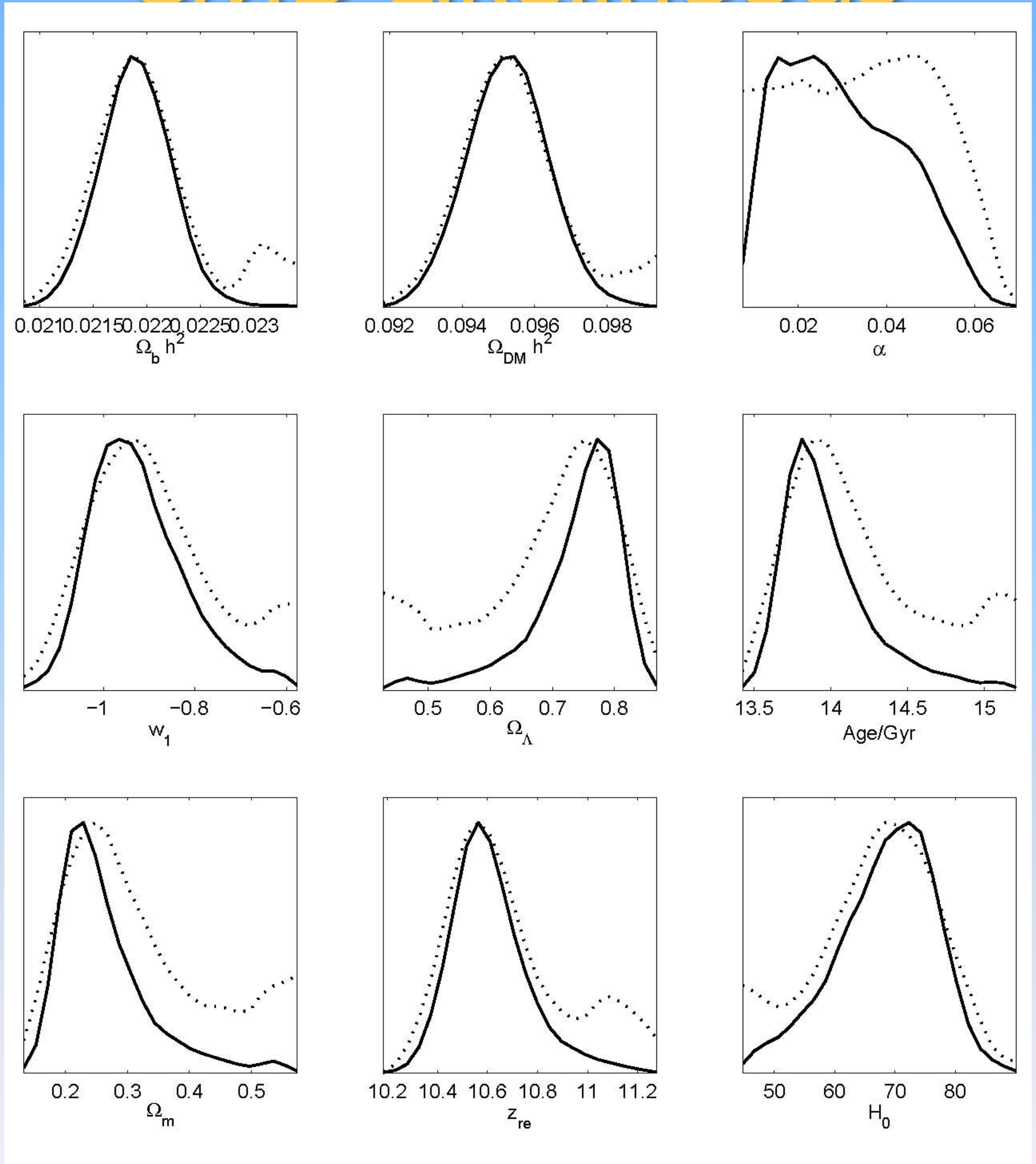


Footprint of Power-law Quintessence model on the CMB and Large Scale Structure

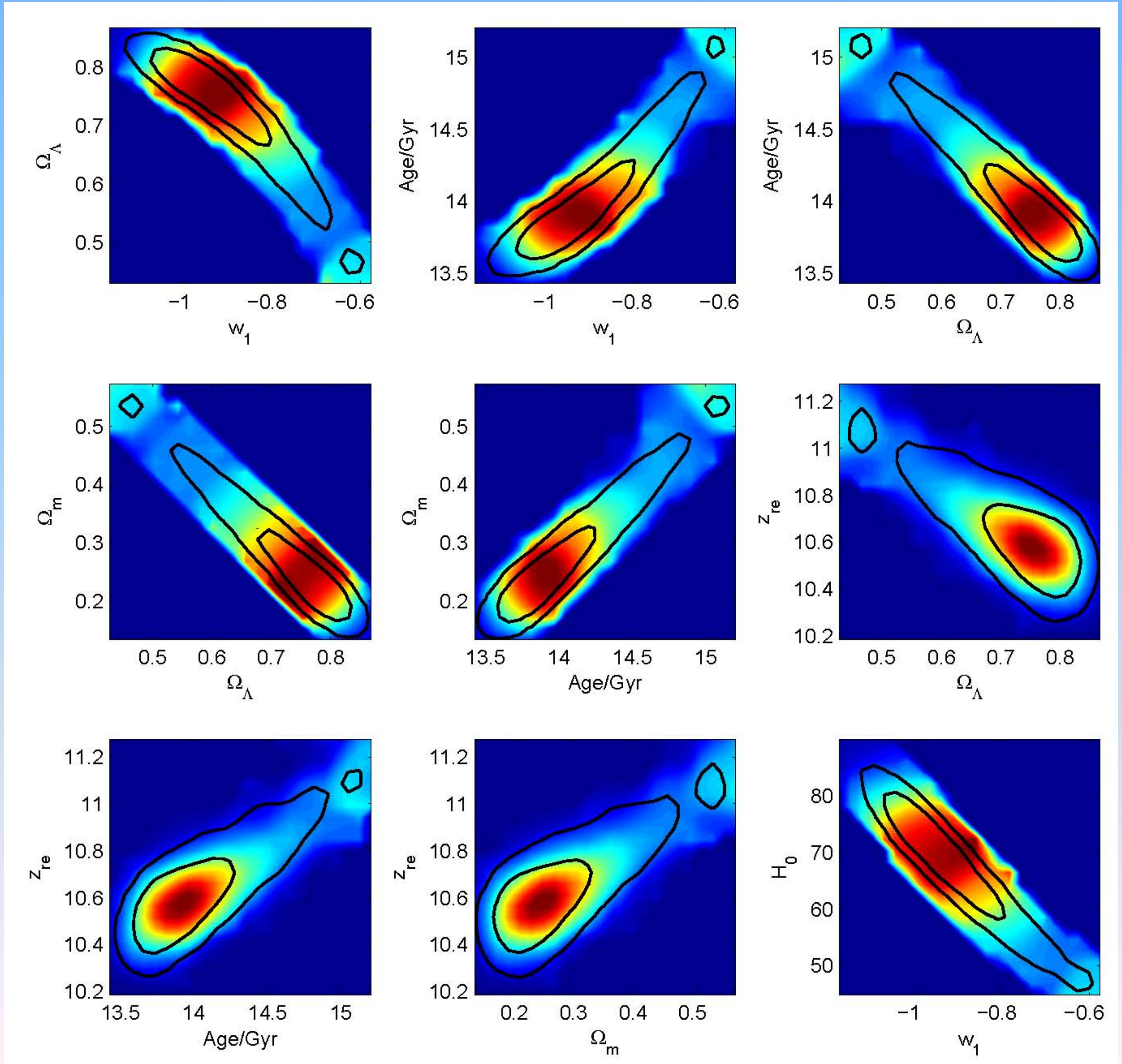
Walking into the parameter space

- A lot of parameter
- Impossibility of exact solution: Monte Carlo Markov Chains
- MCMC in cosmological parameter space: Open source codes: CosmoMC http://cosmologist.info/cosmomc
- Modification of CosmoMC in right way
- Getting best results

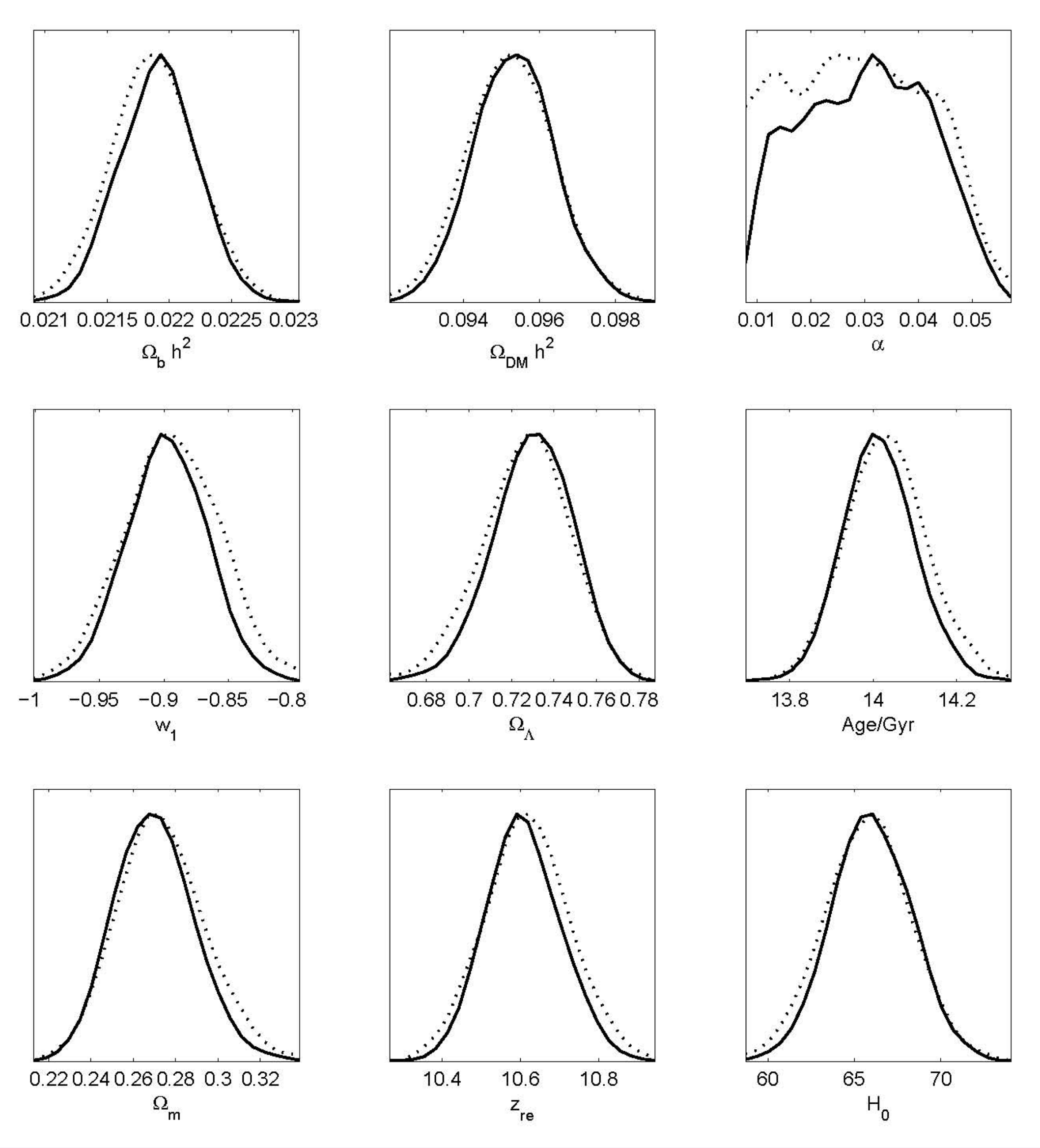
CMB-Likelihoods



CMB-Observational Constraints

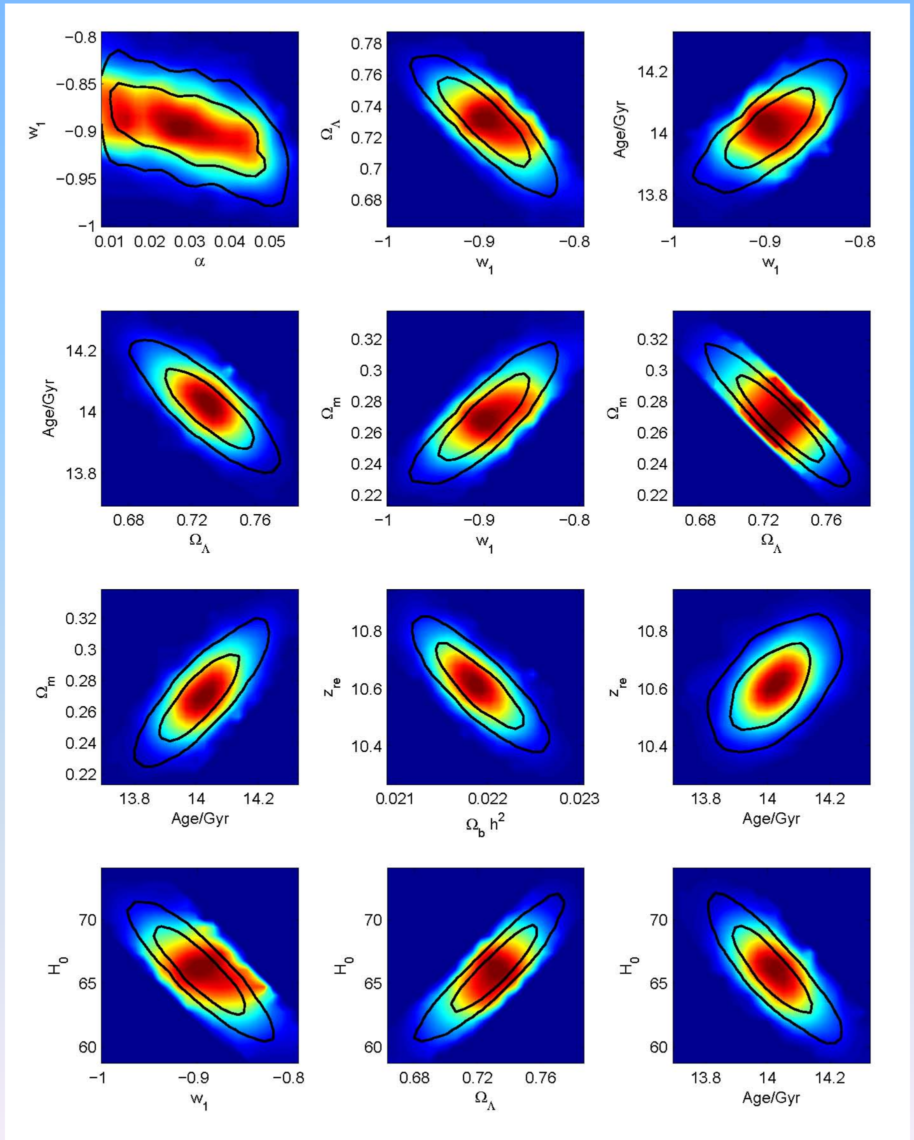


CMB+BAO-Likelihoods



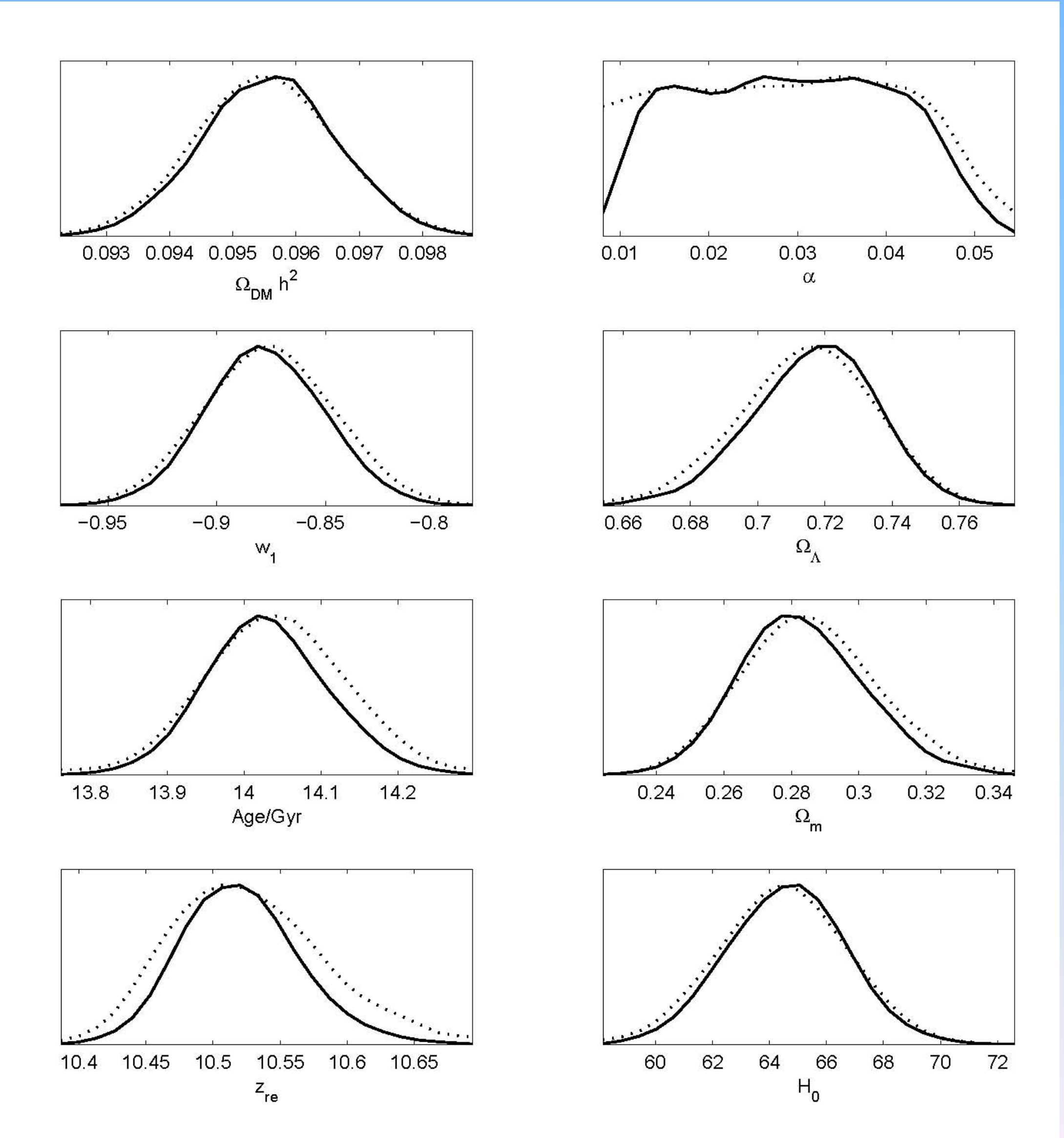
Footprint of Power-law Quintessence model on the CMB and Large Scale Structure

CMB+BAO-Observational Constraints

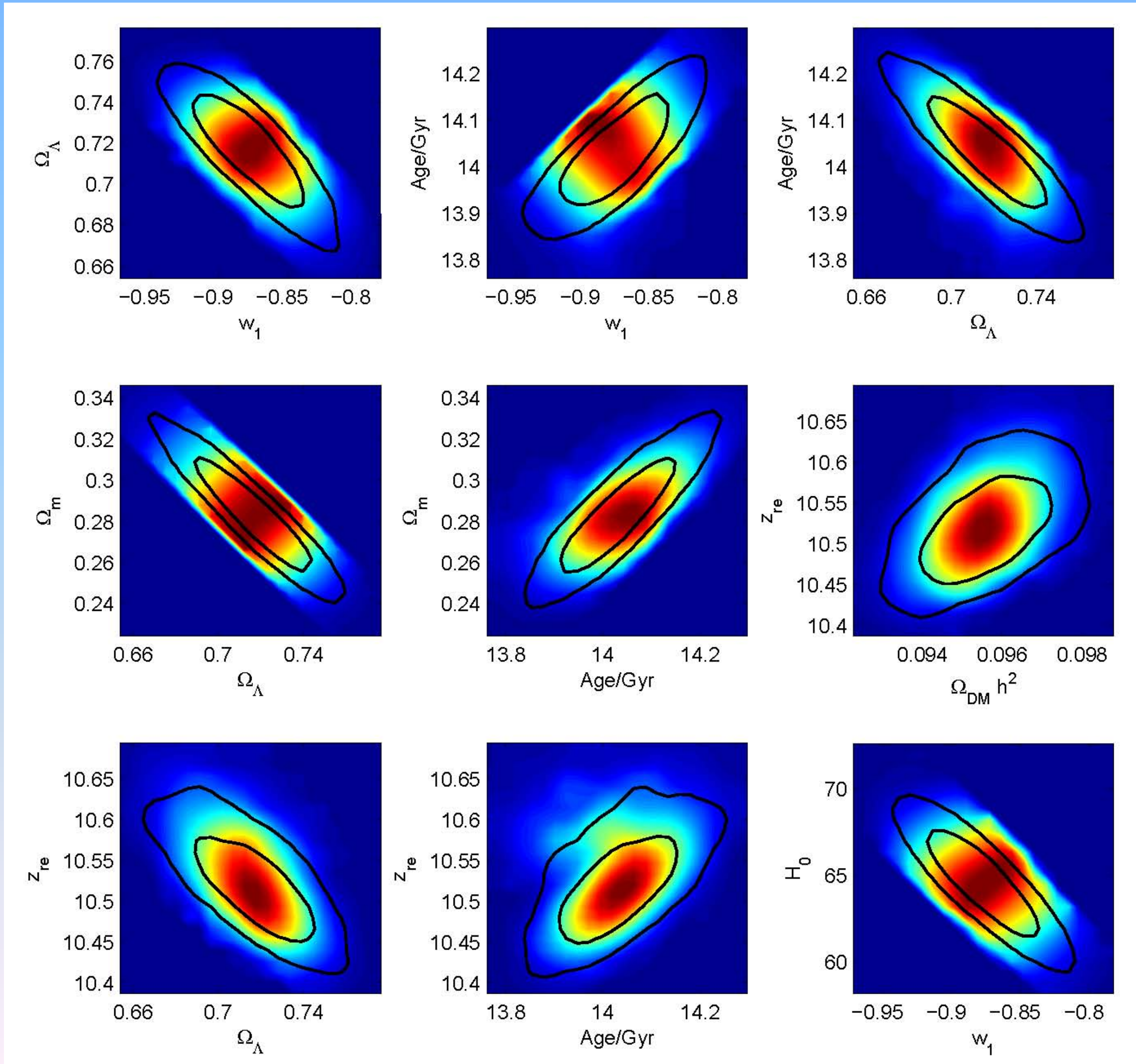


Footprint of Power-law Quintessence model on the CMB and Large Scale Structure

CMB+SN-Likelihoods



CMB+SN -Observational Constraints



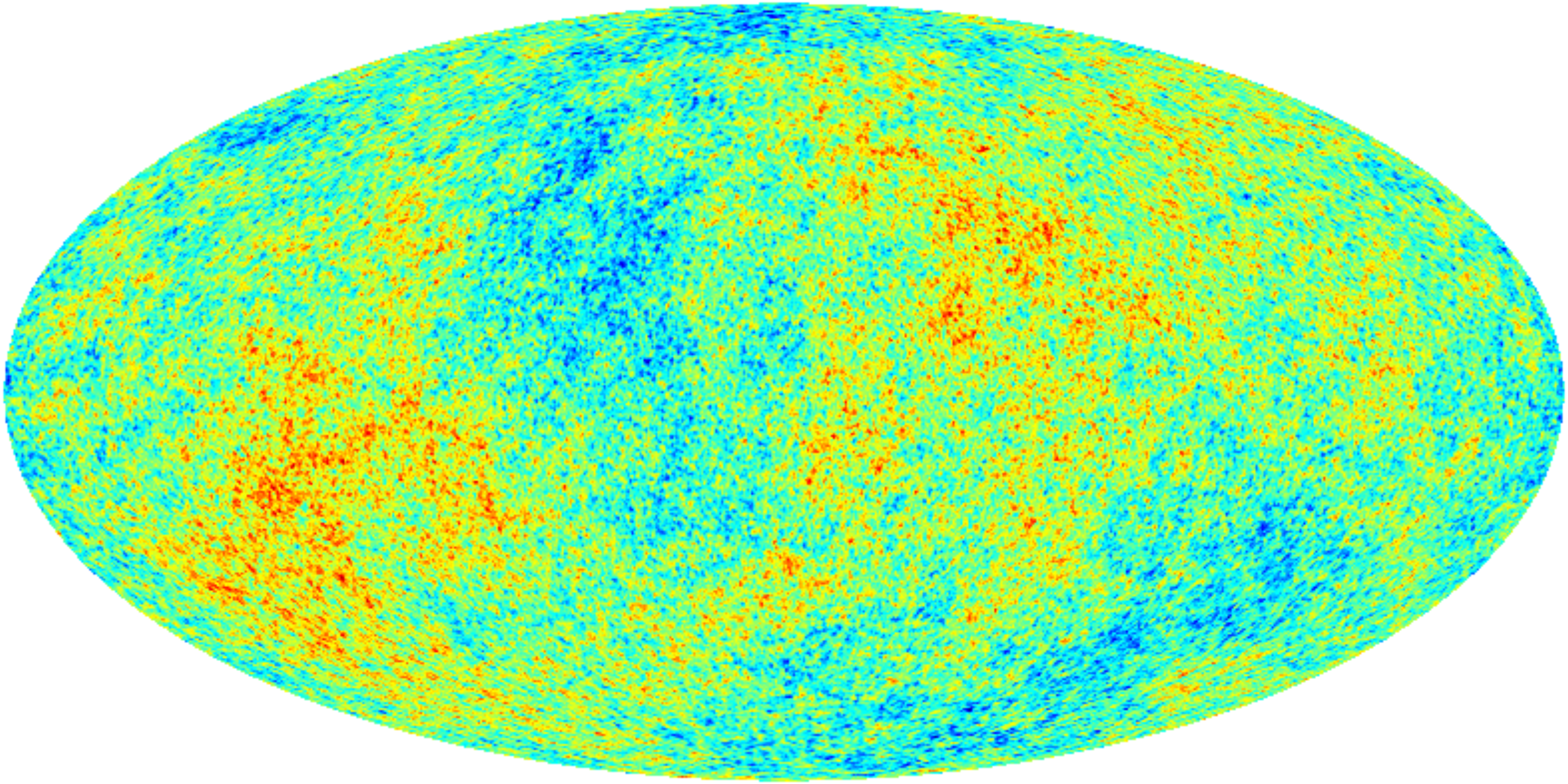
Constraints

Parameter	CMB	CMB+SN
$\Omega_b h^2$	$0.0219^{+0.0006}_{-0.0003}$	0.0223
$\Omega_{cdm} h^2$	$0.0952^{+0.0010}_{-0.0010}$	$0.0955^{+0.001}_{-0.001}$
α	$0.030^{+0.015}_{-0.020}$	$0.029^{+0.012}_{-0.013}$
w_0	$-0.921^{+0.100}_{-0.096}$	$-0.877^{+0.026}_{-0.026}$

Constraints

Parameter	CMB+BAO	CMB+MPK
$\Omega_b h^2$	$0.0219^{+0.0003}_{-0.0003}$	0.0224
$\Omega_{cdm} h^2$	$0.0953^{+0.0010}_{-0.0010}$	0.1161
α	$0.029^{+0.012}_{-0.019}$	$0.021^{+0.006}_{-0.006}$
w_0	$-0.897^{+0.030}_{-0.030}$	$-0.648^{+0.050}_{-0.032}$
σ_8		$0.572^{+0.016}_{-0.020}$

Temperature Anisotropy Map for Power-law



Conclusion and Summary

- Observable quantities
- CMB physics
- Importance of Power Spectrum: Primordial, Temperature and matter power spectrum
- Observational Constraints