In the name of Allah

Cosmological Codes: An Introduction to Computational Cosmology

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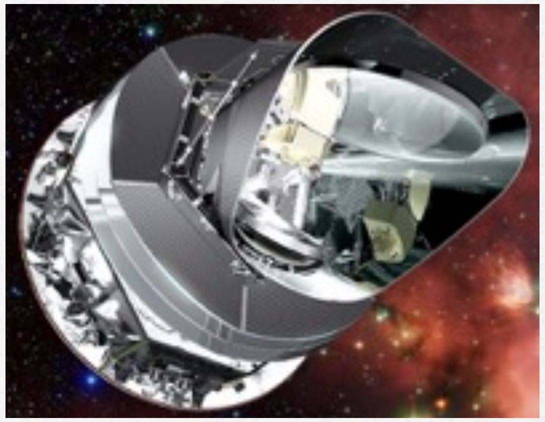
Outline:

- Why we use need computer codes?
- Classification of Cosmological Codes
- CAMB code
- CosmoMC code

Why computer codes?

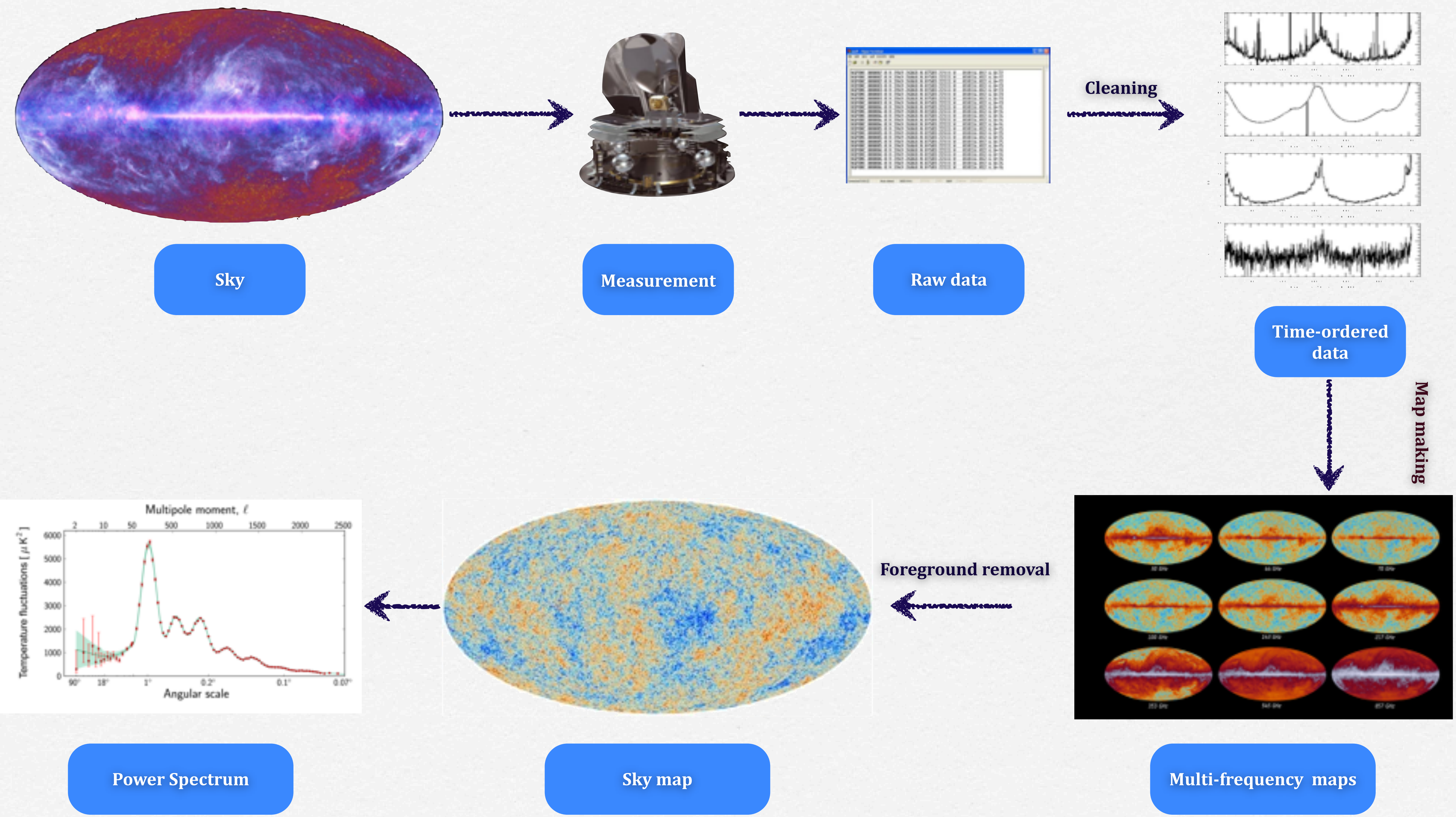
- Complexity of calculations
- Using observational data
- Time is valuable !



Classification

- Map generation and processing codes
- Boltzmann codes
- Parameter estimator codes

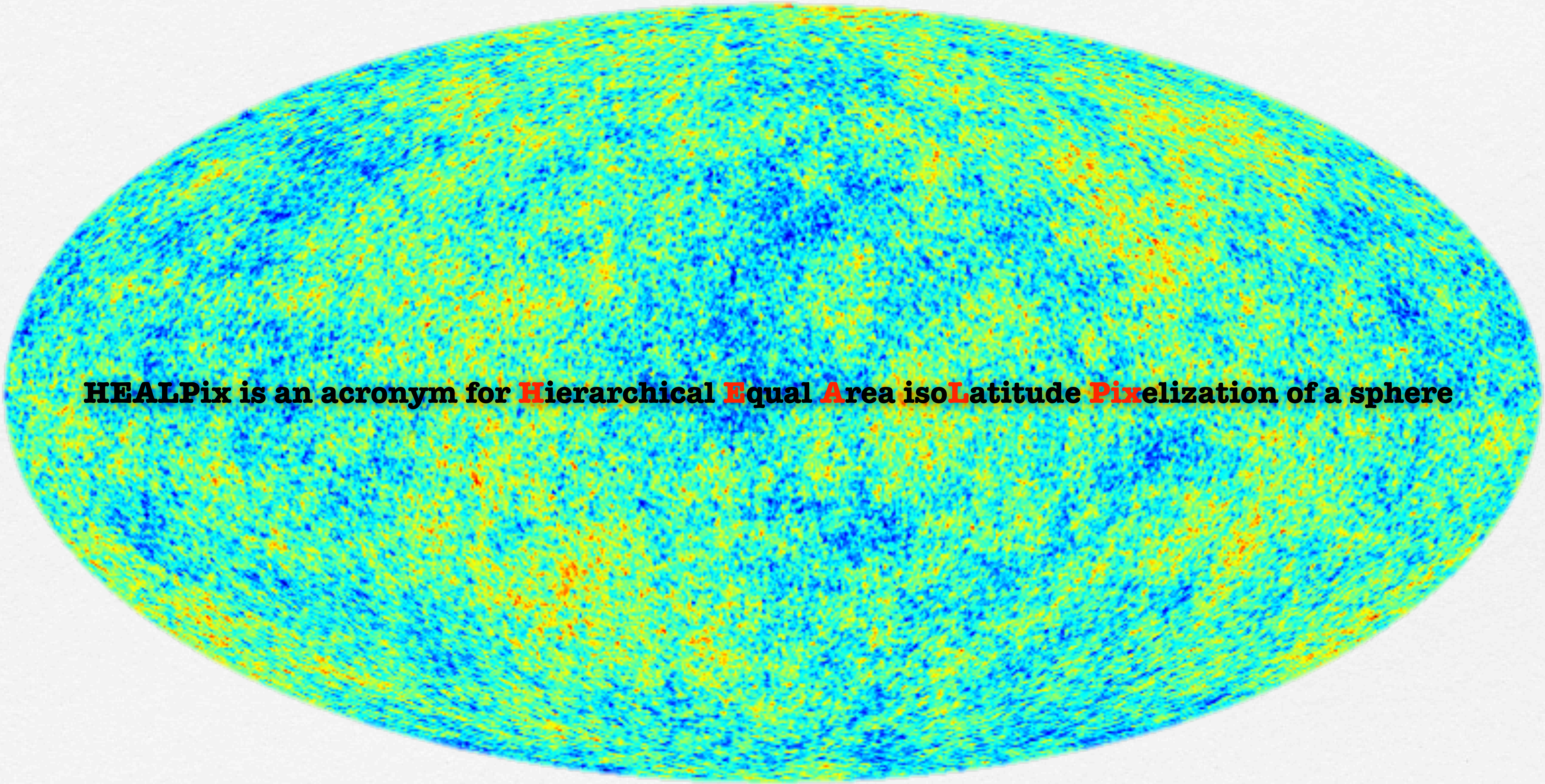
CMB DATA ANALYSIS



Map generation and analysis

- HEALPix
- GLESP
- Commander 2
- WeightMixer

HEALPix



HEALPix is an acronym for Hierarchical Equal Area isoLatitude Pixelization of a sphere

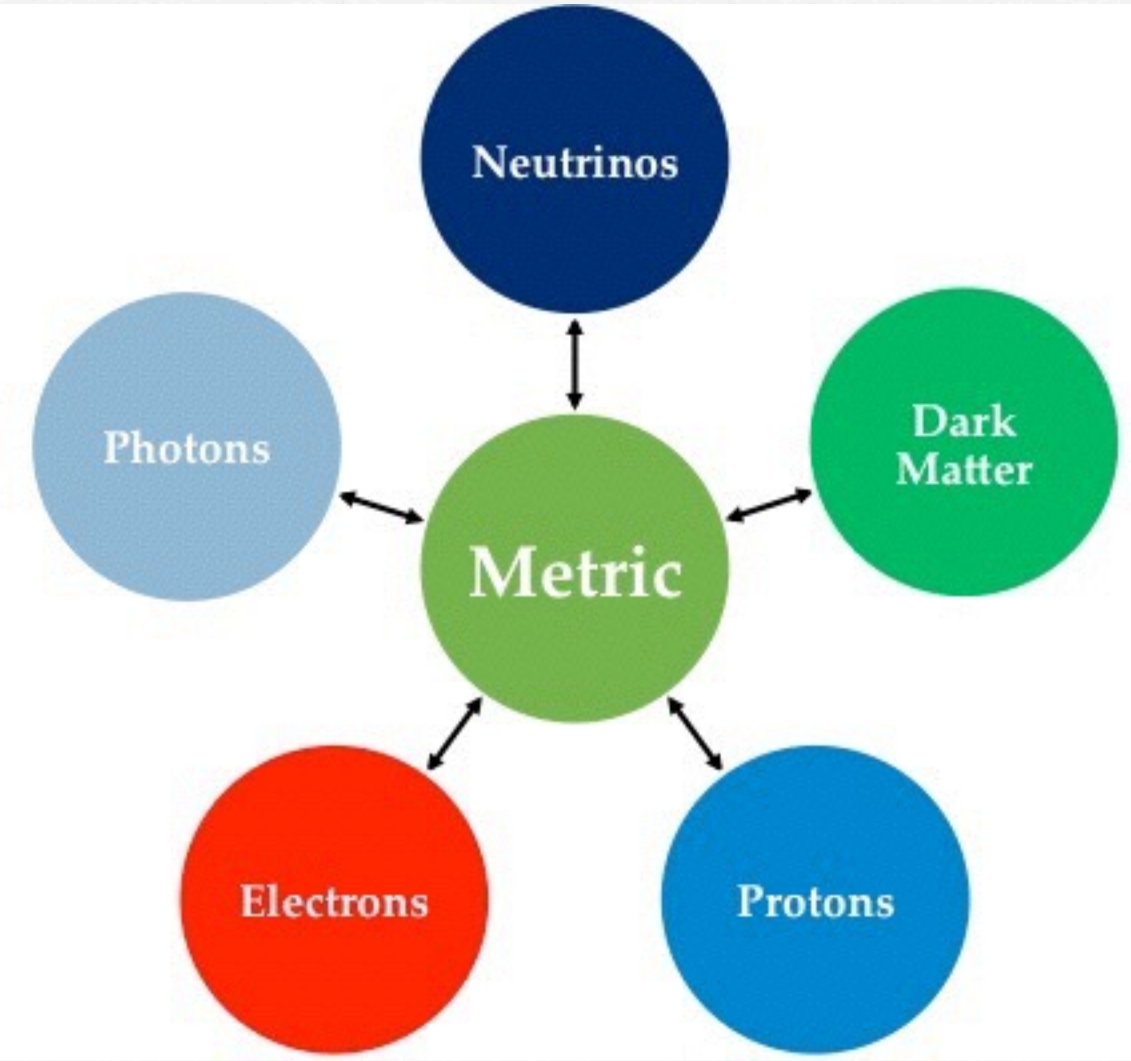
Boltzmann codes

- CAMB
- CMBEASY
- CLASS II
- CMBAns
- CosmoLib
- RECFAST

INGREDIENTS FOR COSMIC SOUP

Boltzmann equation

$$\boxed{\frac{df}{dt} = C[f]}$$



Required physics

Einstein equ.
- Ψ Newtonian potential
- Φ Spatial curvature

Boltzmann equ.
- Θ_T Photon temperature fluctuation
- Θ_P Photon polarization fluctuation

Euler/continuity equ.
- δ_b, δ_c Baryon, CDM density fluctuation
- v_b, v_c Baryon, CDM velocity fluctuation

Einstein-Boltzmann equations

Liouville term ⟷ **Collision term**

Einstein+Boltzmann equ.

1) $\dot{\Theta}_T + ik\mu\Theta_T + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi]$

2) $\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}[-\Theta_P + \frac{1}{2}(1-\mathcal{P}_2(\mu))\Pi]$

$\Pi = \Theta_{T2} + \Theta_{P2} + \Theta_{P0}$

3) $\dot{\mathcal{N}} + ik\mu\mathcal{N} + \dot{\Phi} + ik\mu\Psi = 0$

Photon/Neutrinos

EB equs. with fluid approximation (Euiler+continuity)

4) $\dot{\delta}_c = -ikv_c - 3\dot{\Phi}$

5) $\dot{\delta}_b = -ikv_b - 3\dot{\Phi}$

6) $\dot{v}_c = -Hv_c - ik\Psi$

7) $\dot{v}_b = -Hv_b - ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1], \quad \frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$

Baryon/CDM

Einstein-Boltzmann equations

Perturbed metric $$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)dx^2$$

$$\begin{cases} 8)\ k^2\Phi + 3\frac{\dot{a}}{a}(\dot{\Phi} - \Psi\frac{\dot{a}}{a}) = 4\pi G a^2[\rho_{CDM}\delta + \rho_b\delta_b + 4(\rho_\gamma\Theta_0 + \rho_\nu\mathcal{N}_0)] \\ 9)\ k^2(\Phi + \Psi) = -32\pi G a^2(\rho_\gamma\Theta_2 + \rho_\nu\mathcal{N}_2) \end{cases}$$

Gravity

Initial condition

astro-ph/0606683

$$\Theta_0 = \frac{1}{2}\Phi\,,$$

$$\delta = \delta_b = \frac{3}{2}\Phi\,,$$

$$\Theta_1 = -\frac{k}{6\mathcal{H}}\Phi\,,$$

$$v = v_b = \frac{k}{2\mathcal{H}}\Phi\,.$$

$$3\Theta_1 + v_b = 0$$

$$P_\Phi(k) = \frac{8\pi}{9k^3}\frac{H^2}{\epsilon m_{pl}^2}\bigg|_{aH=k} \equiv \frac{50\pi^2}{9k^3}\left(\frac{k}{H_0}\right)^{n-1}\delta_H^2\left(\frac{\Omega_m}{D_1(a=1)}\right)^2$$

Hierarchy equations

$$\Theta_0' = -\frac{k}{\mathcal{H}}\Theta_1 - \Phi'$$

$$\Theta_1' = \frac{k}{3\mathcal{H}}\Theta_0 - \frac{2k}{3\mathcal{H}}\Theta_2 + \frac{k}{3\mathcal{H}}\Psi + \tau'\left[\Theta_1 + \frac{1}{3}v_b\right]$$

$$\Theta_l' = \frac{lk}{(2l+1)\mathcal{H}}\Theta_{l-1} - \frac{(l+1)k}{(2l+1)\mathcal{H}}\Theta_{l+1} + \tau'\left[\Theta_l - \frac{1}{10}\Pi\delta_{l,2}\right], \quad (l \geq 2)$$

$$\Theta_{P0}' = -\frac{k}{\mathcal{H}}\Theta_1^P + \tau'\left[\Theta_0^P - \frac{1}{2}\Pi\right]$$

$$\Theta_{Pl}' = \frac{lk}{(2l+1)\mathcal{H}}\Theta_{l-1}^P - \frac{(l+1)k}{(2l+1)\mathcal{H}}\Theta_{l+1}^P + \tau'\left[\Theta_l^P - \frac{1}{10}\Pi\delta_{l,2}\right], \quad (l \geq 1)$$

Hierarchy equations

$$\mathcal{N}_0' = -\frac{k}{\mathcal{H}}\mathcal{N}_1 - \Phi'$$

$$\mathcal{N}_1' = \frac{k}{3\mathcal{H}}\mathcal{N}_0 - \frac{2k}{3\mathcal{H}}\mathcal{N}_2 + \frac{k}{3\mathcal{H}}\Psi$$

$$\mathcal{N}_1' = \frac{lk}{(2l+1)\mathcal{H}}\mathcal{N}_{l-1} - \frac{(l+1)k}{(2l+1)\mathcal{H}}\mathcal{N}_{l+1}, \quad (l \geq 2)$$

$$\delta' = \frac{k}{\mathcal{H}}v - 3\Phi'$$

$$v' = -v - \frac{k}{\mathcal{H}}\Psi$$

$$\delta_b' = \frac{k}{\mathcal{H}}v_b - 3\Phi'$$

$$v_b' = -v_b - \frac{k}{\mathcal{H}}\Psi + \tau' R(3\Theta_1 + v_b)$$

$$\Phi' = \Psi - \frac{k^2}{3\mathcal{H}^2}\Phi + \frac{H_0^2}{2\mathcal{H}^2}\left[\Omega_m a^{-1}\delta + \Omega_b a^{-1}\delta_b + 4\Omega_r a^{-2}\Theta_0 + 4\Omega_\nu a^{-2}\mathcal{N}_0\right]$$

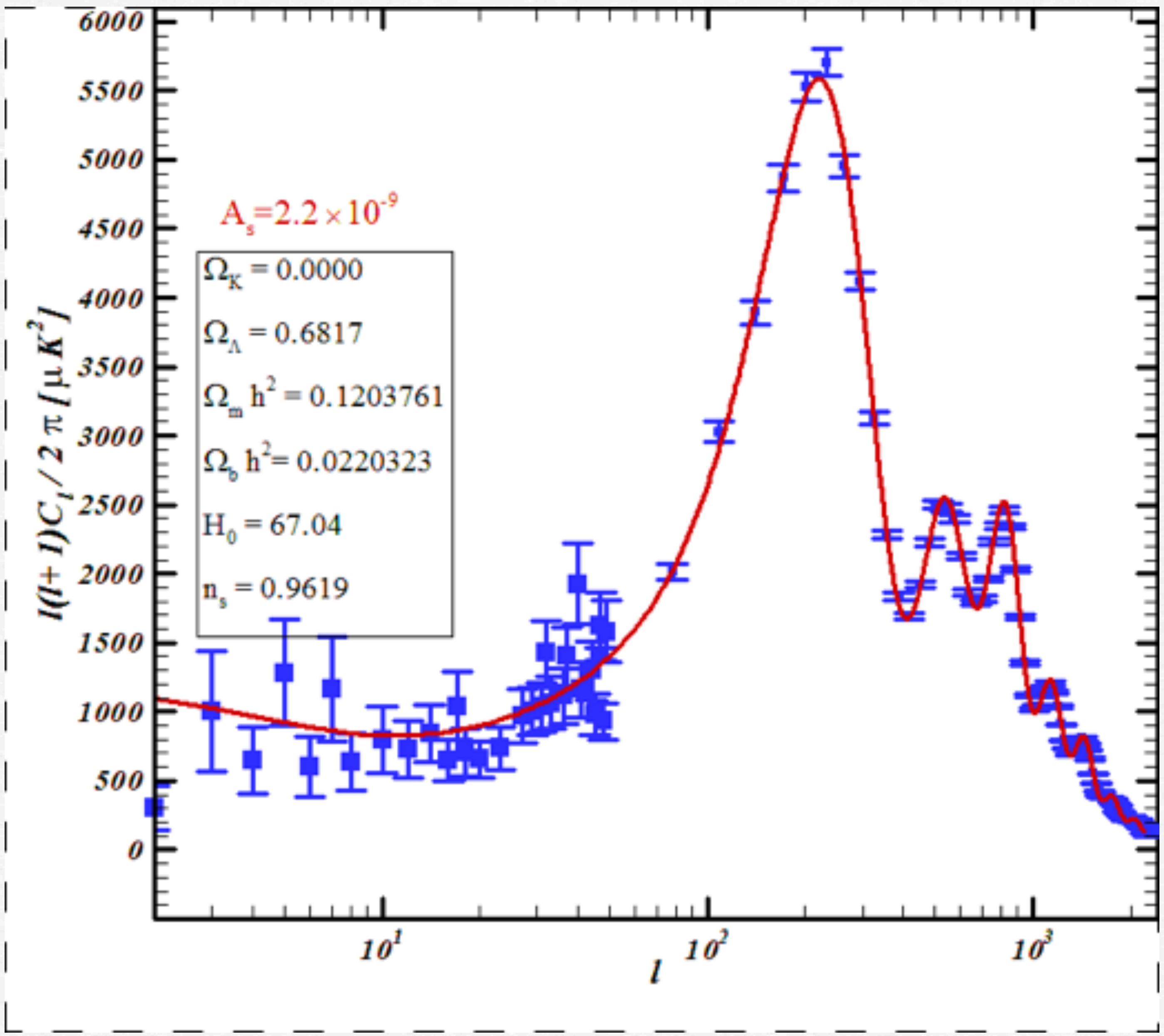
$$\Psi = -\Phi - \frac{12H_0^2}{k^2a^2}\left[\Omega_r\Theta_2 + \Omega_\nu\mathcal{N}_2\right]$$

CAMB

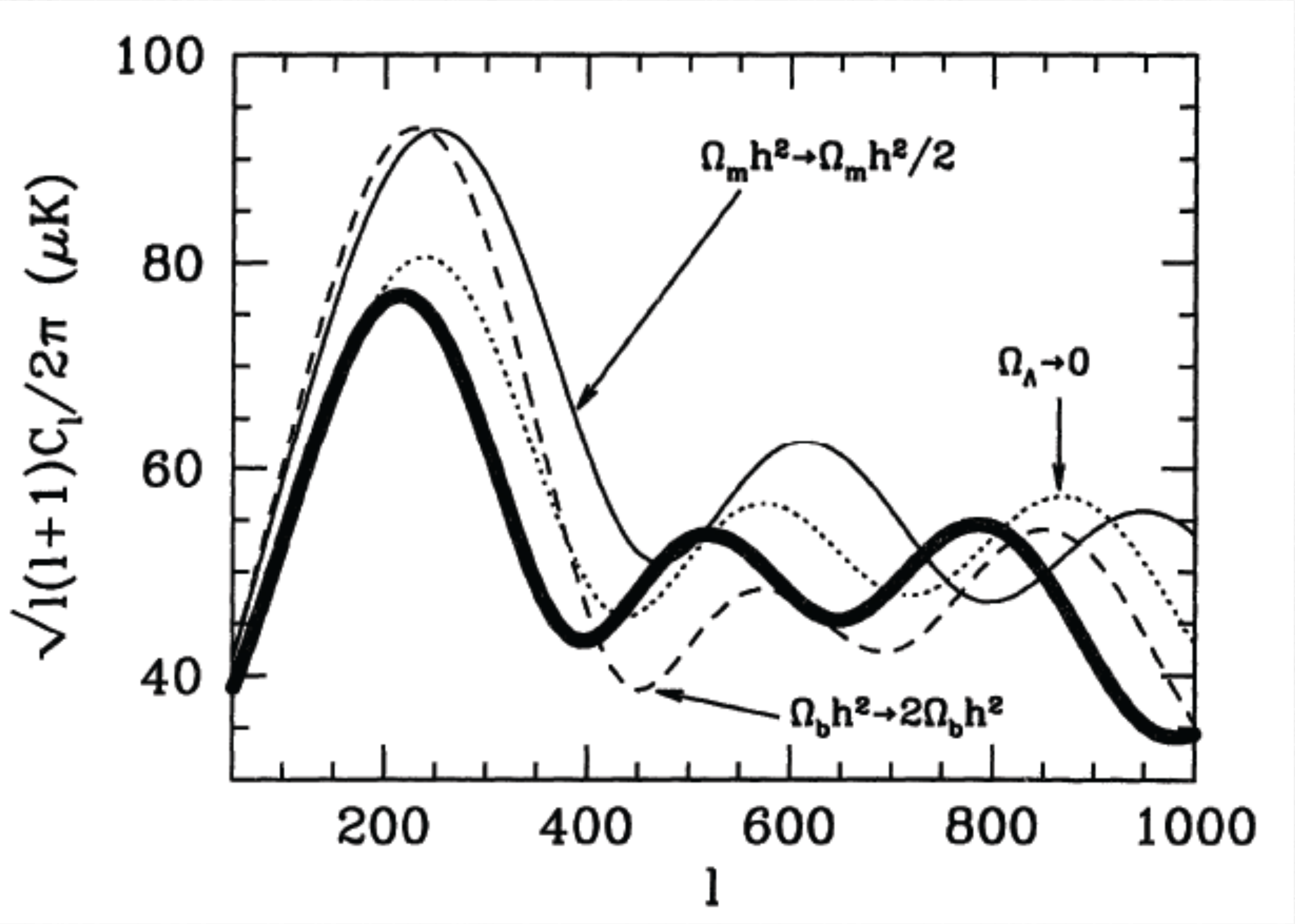
- Code for Anisotropies in the Microwave Background
- Temperature power spectrum
- Curved and flat models
- Matter power spectrum
- Massive neutrino models
- Scalar, Tensor and vector perturbations

$$l,\ C_{TT},\ C_{EE},\ C_{BB},\ C_{TE}$$

Temperature power spectrum



PARAMETERS EFFECTS

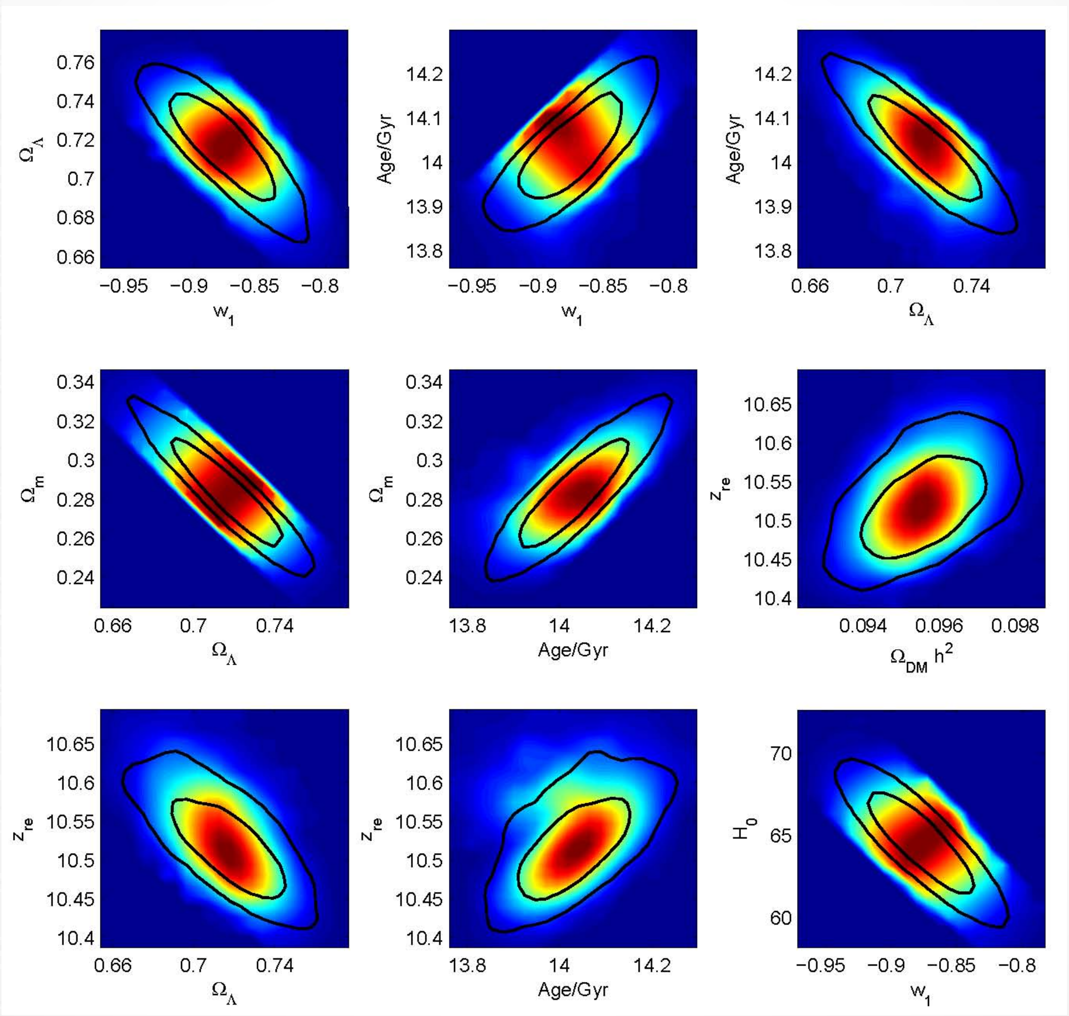


S.Dodelson,"Modern Cosmology"

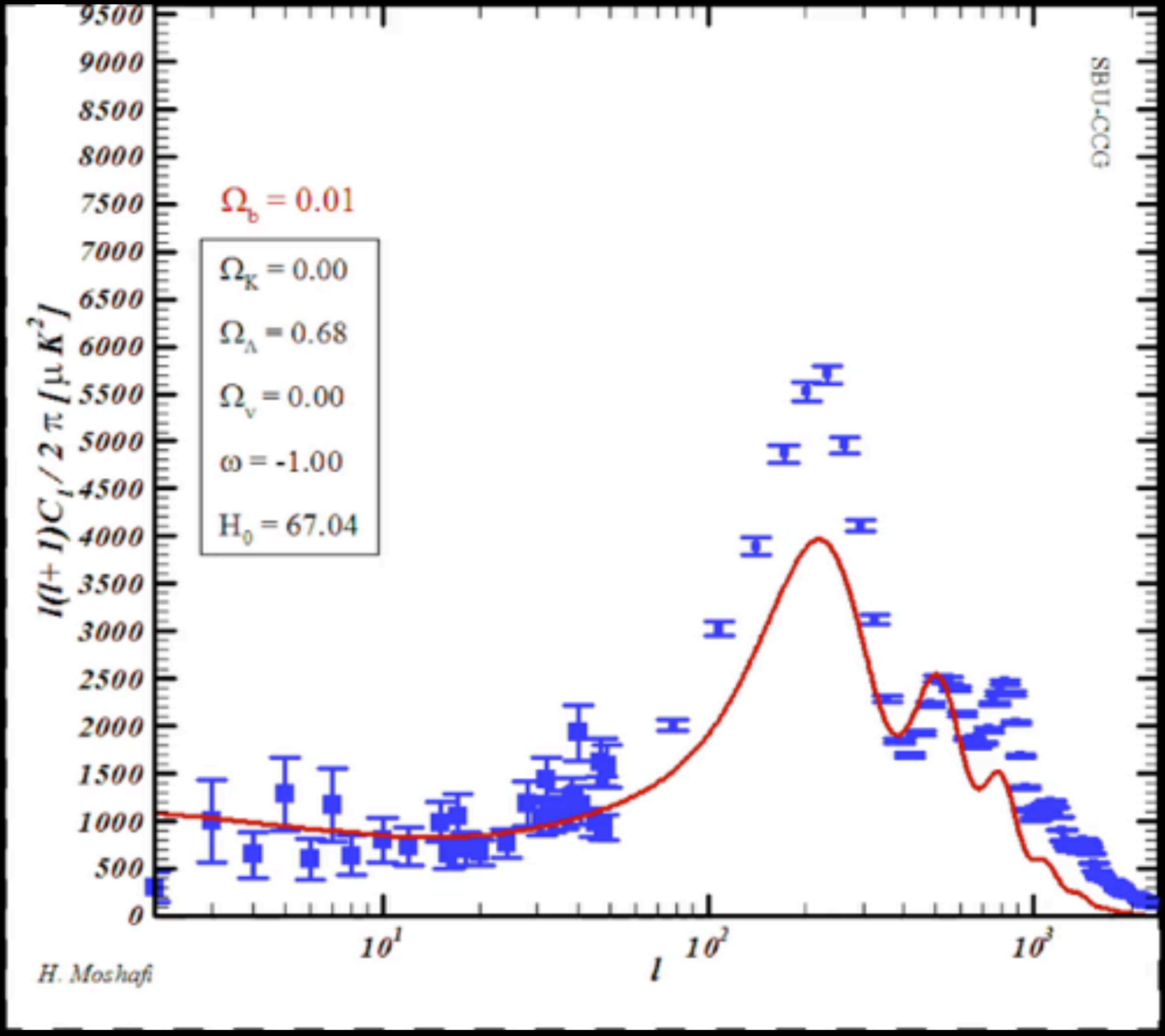
Cosmological parameter estimator

- CosmoMC
- AnalyzeThis
- SCoPE

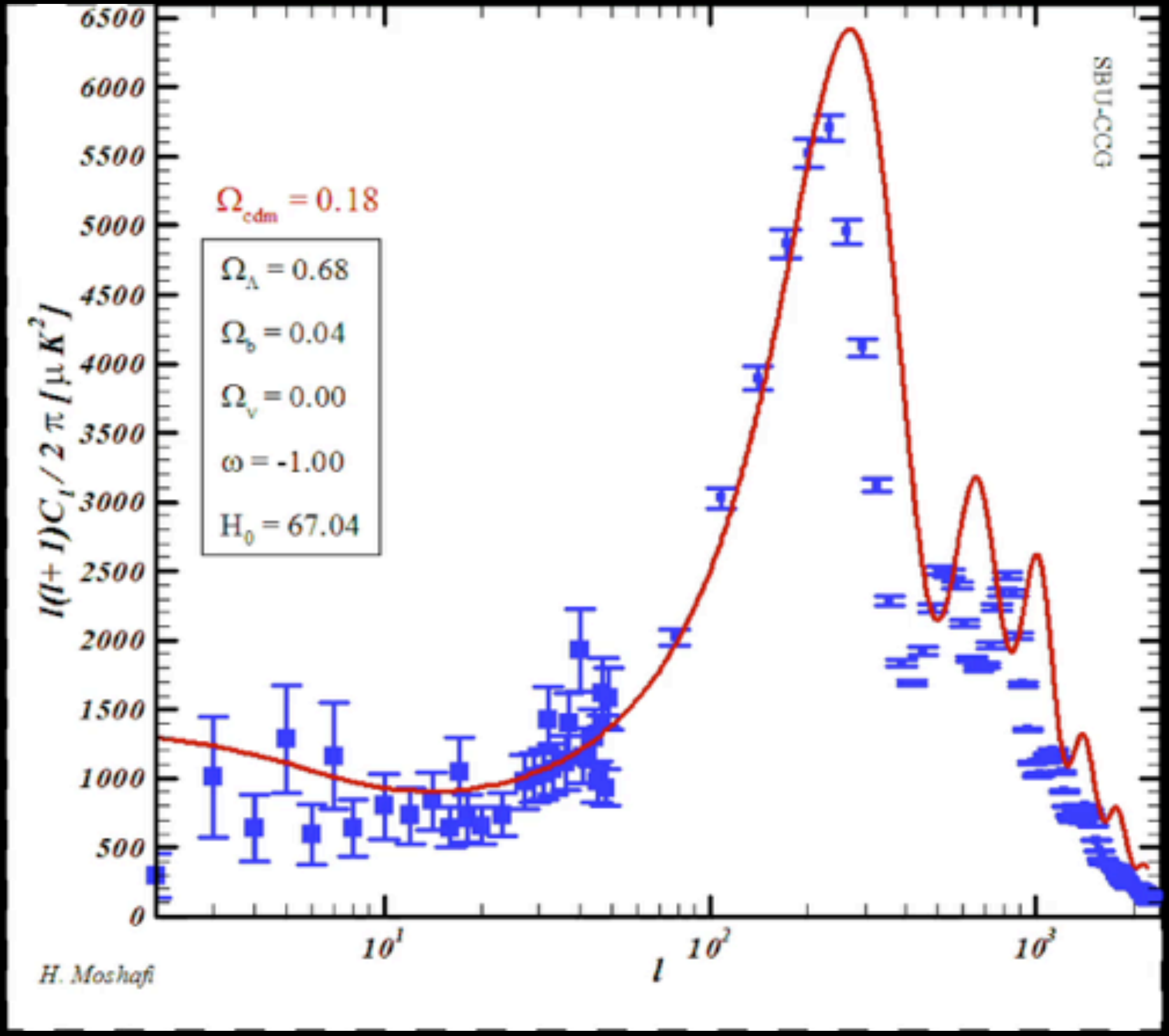
CMB+SN-contours



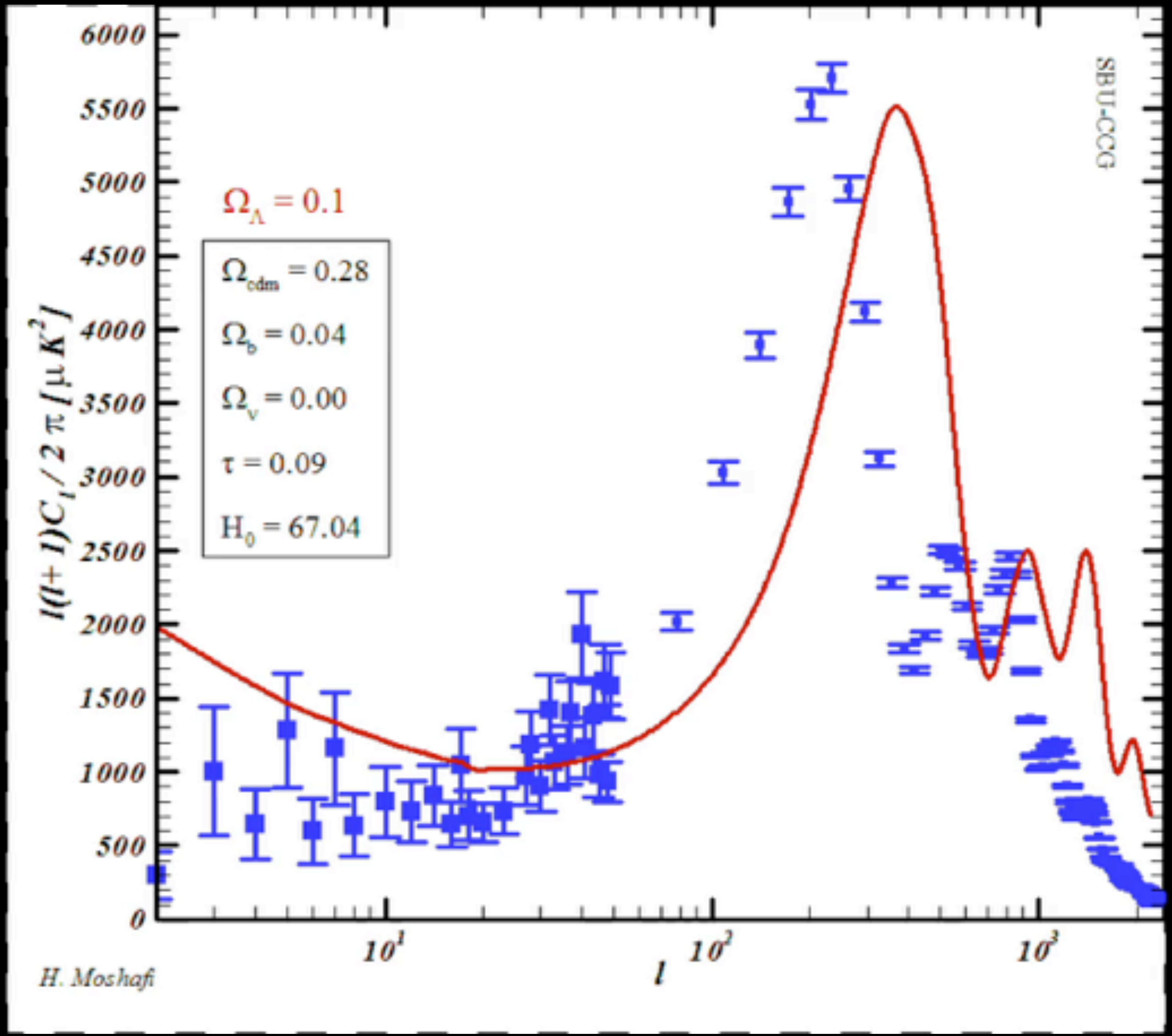
Power Spectrum



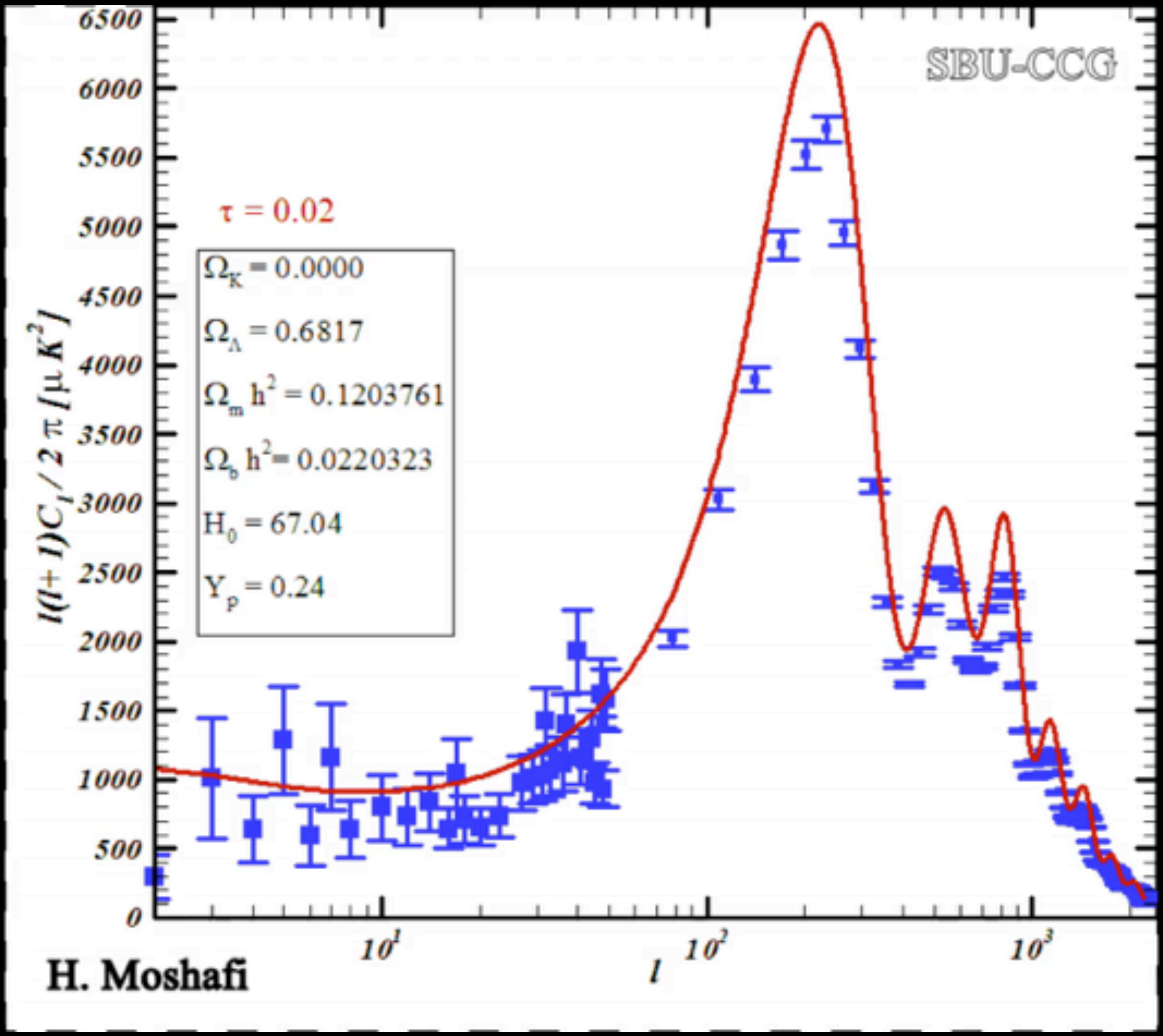
Power Spectrum



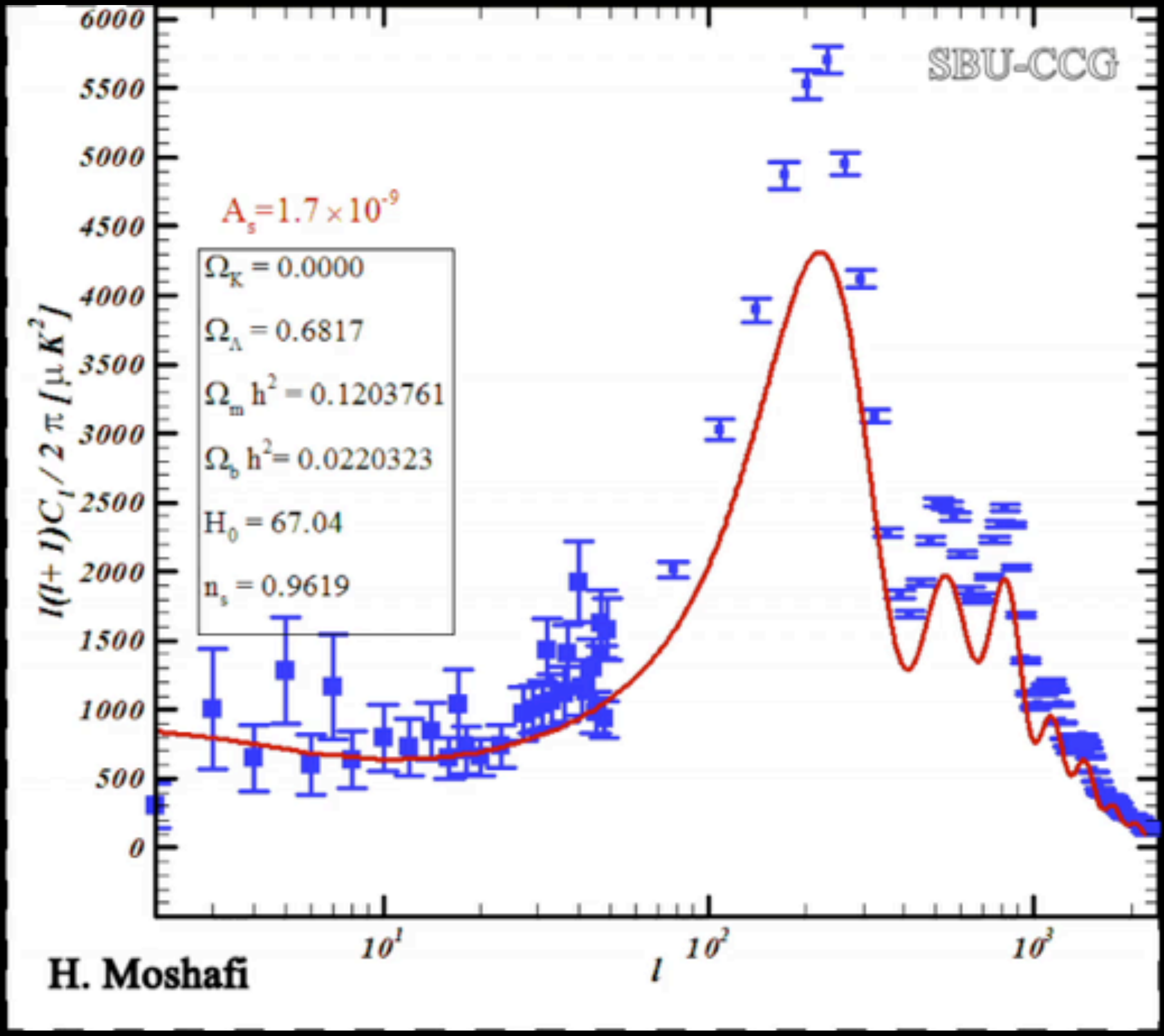
Power Spectrum



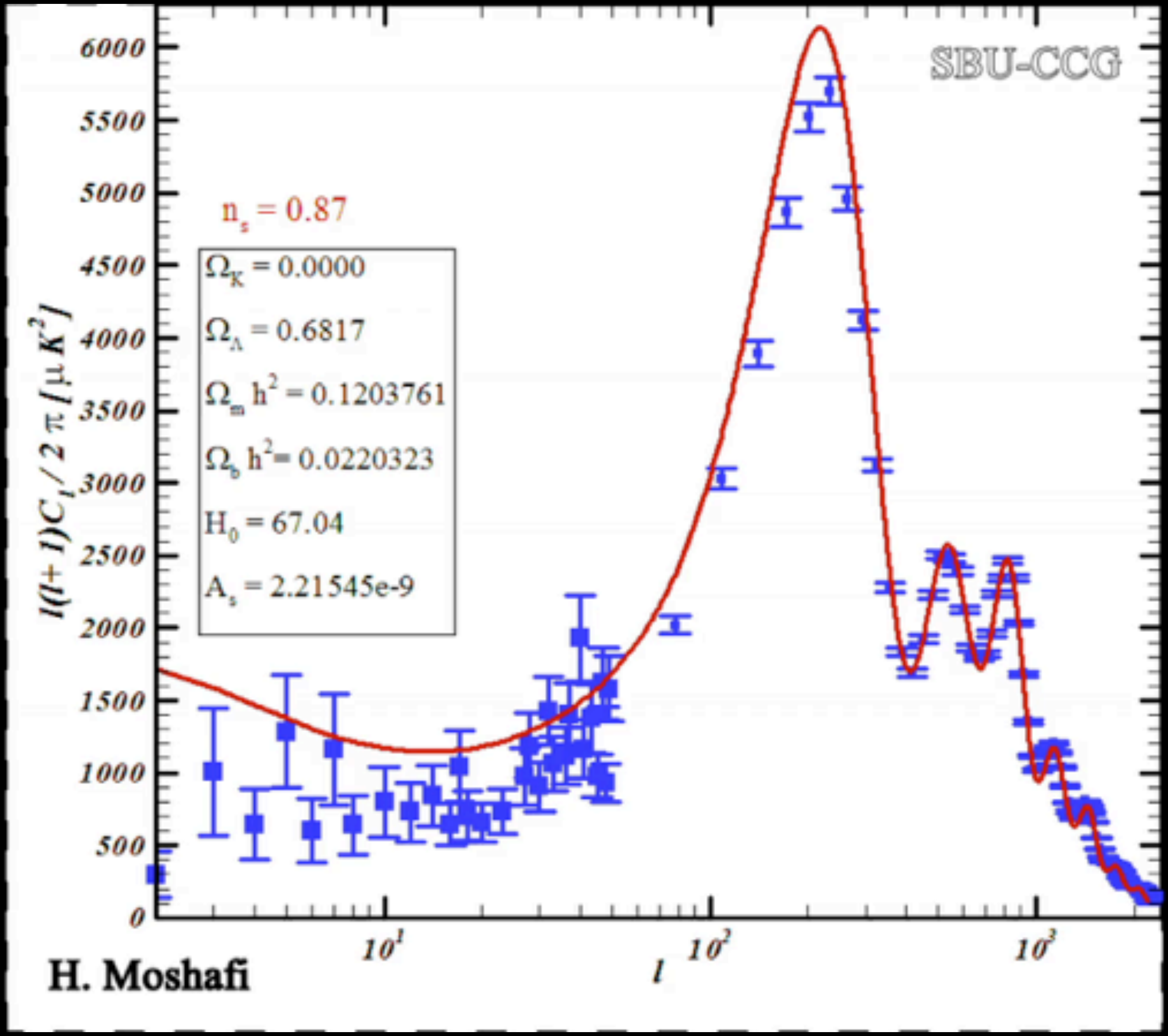
Power Spectrum



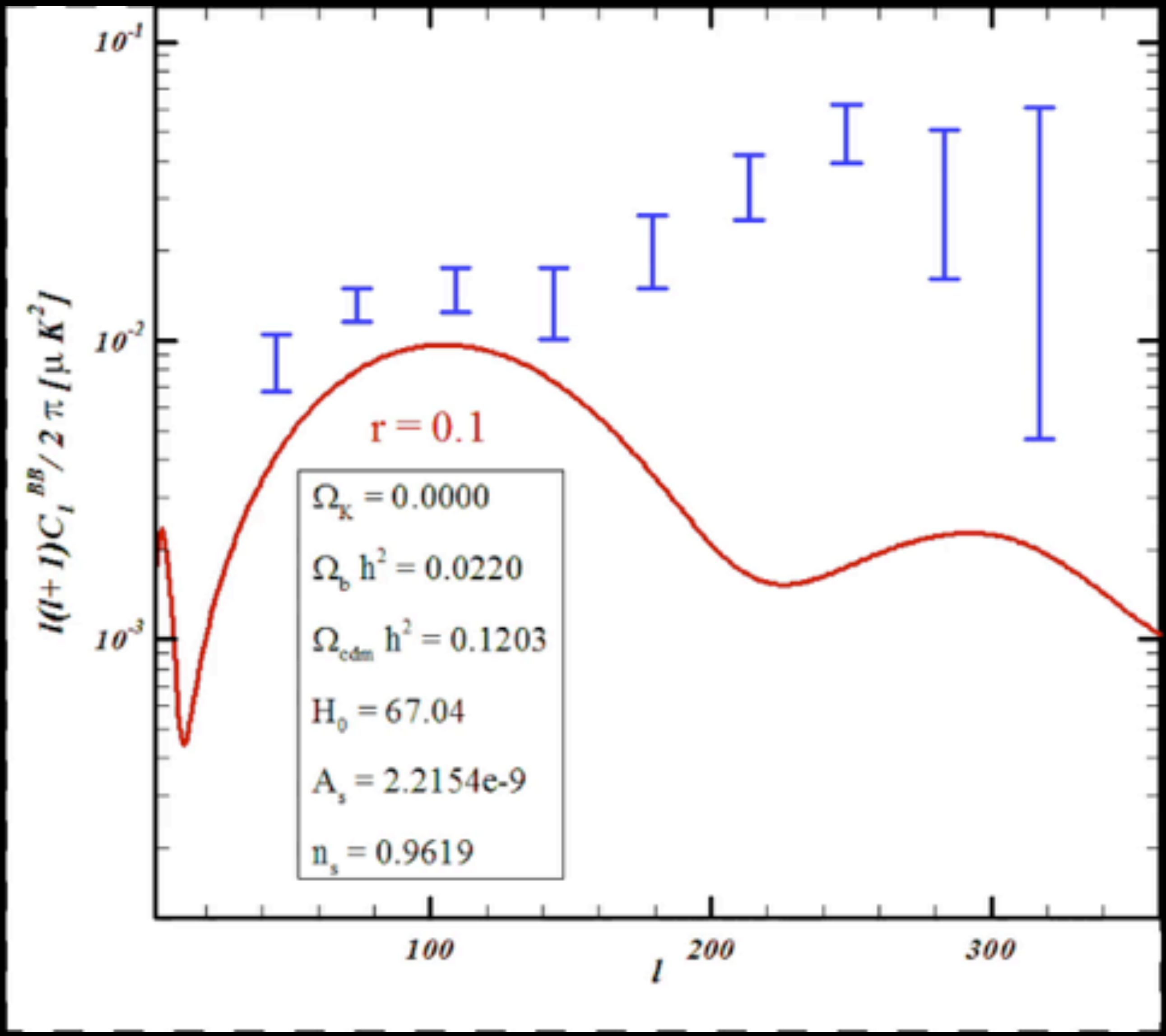
Power Spectrum



Power Spectrum



Power Spectrum



COSMOLOGICAL MONTECARLO

CosmoMC is a Fortran 2008 Markov-Chain Monte-Carlo (MCMC) engine for exploring cosmological parameter space, together with Fortran and python code for analyzing Monte-Carlo samples and importance sampling. The code does brute force (but accurate) theoretical matter power spectrum and C_l calculations with CAMB.

Pubic Code: http://cosmologist.info/cosmomc/

Maximum-Likelihood Estimation

- Method of estimating the parameters of a model