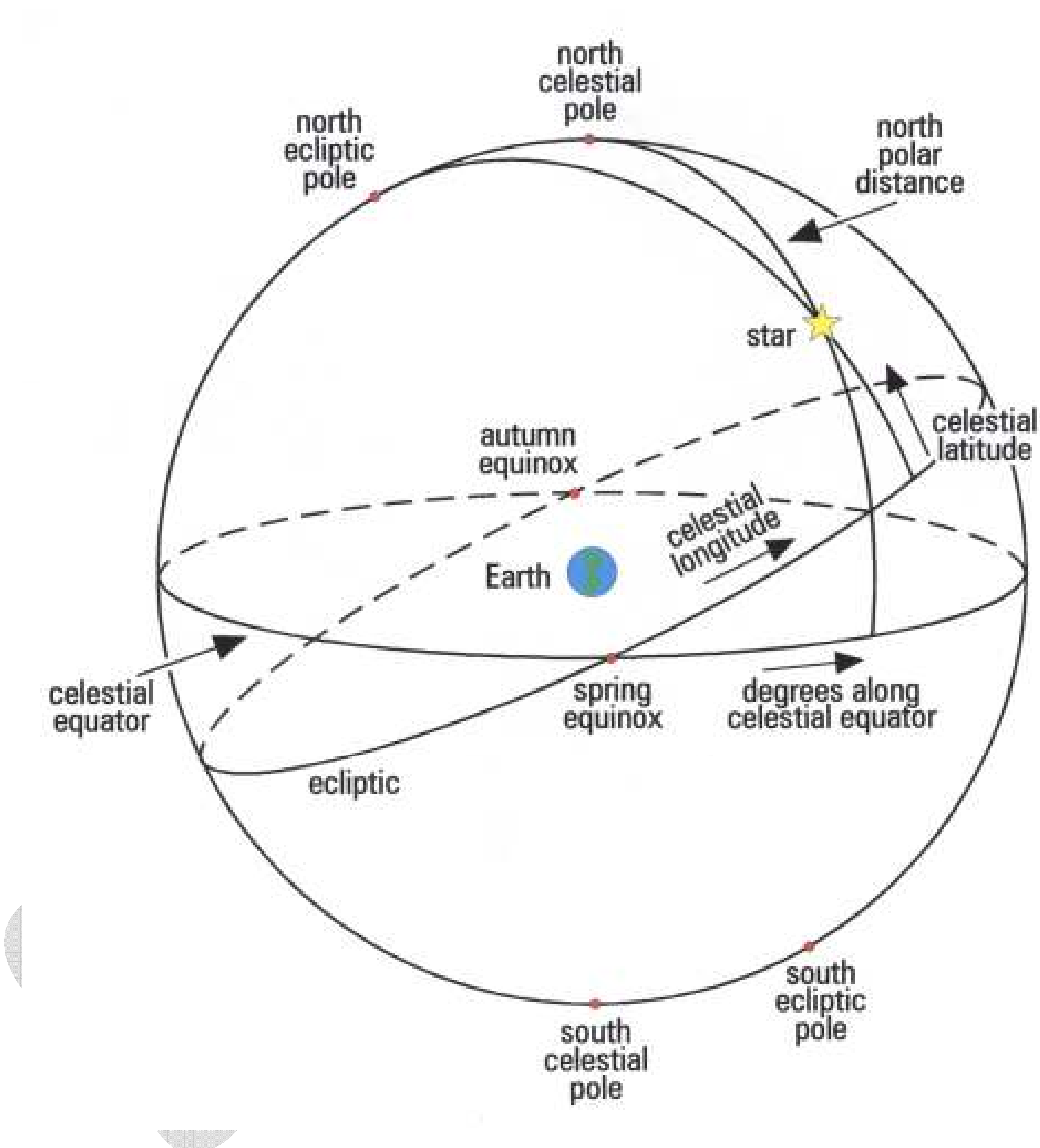
حل مسائل فصل دوم کتاب نجوم کروی



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Solution 1

$cos\ z_1 = \sin\phi\sin\delta + \cos\phi\cos\delta \Rightarrow \cos z_1 = \cos\ (\phi-\delta) \Rightarrow \sin z_1 = \sin(\phi-\delta) = \sin\phi\cos\delta - \cos\phi\sin\delta$

$\sin\delta = \sin\phi\cos z + \cos\phi\sin z\cos A \overset{A=90°}{\Rightarrow} \sin\delta = \sin\phi\cos z_2 \Leftrightarrow \sin\phi = \dfrac{\sin\delta}{\cos z_2}$

$\sin z_1 \sin\delta + \cos z_1\cos\delta = \sin\phi = \dfrac{\sin\delta}{\cos z_2} \xrightarrow{deviding\ by\ (\sin\delta)} \boxed{\cot\delta = \csc z_1\sec z_2 - \cot z_1}$

$\cos z_1\sin\phi - \sin z_1\cos\phi = \sin\delta = \sin\phi\cos z_2 \xrightarrow{deviding\ by\ (\sin\phi)} \boxed{\cot\phi = \cot z_1 - \cos z_2\csc z_1}$

Solution 2 (figure in appendix)

$\dfrac{\sin(90-\psi)}{\sin\phi} = \dfrac{\sin(90°)}{\sin(90°-\delta)} \Rightarrow \boxed{\cos\psi = \sin\phi\sec\delta}$

Solution 3

$\sin\phi\cos H = \cos\phi\tan\delta + \sin H\cos\ (90\ °) \Rightarrow \cos H = \cot\phi\tan\delta\ , \cos h = -\tan\phi\tan\delta \Rightarrow \cos h\cos H = -\tan^2\delta$

$\Rightarrow \boxed{\cos h\cos H + \tan^2\delta = 0}$

$H_{Aldaberan} = 102°.320\ , h_{Aldaberan} = 66°.158 \Rightarrow \Delta T = 2^h24^m39^s\ (siderial\ time) \xrightarrow{\div 1.002738} \boxed{2^h24^m.3 \pm 0^m.1}$

Solution 4 (figure in appendix)

$\Delta r = v\Delta t \Rightarrow \Delta x = -v\Delta t\sin A \equiv -v\ sin\ A\ \Delta H\ , \cos z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \xrightarrow{diffrentiating}$

$\sin z\ \Delta z = \cos\phi\cos\delta\sin H\ \Delta H\ , \dfrac{\sin A}{\cos\delta} = \dfrac{-\sin H}{\sin z} \Rightarrow \Delta H = \dfrac{-\Delta z}{\sin A\cos\phi} \Rightarrow \Delta x = \dfrac{v\ \Delta z}{\cos\phi} = \boxed{\dfrac{5\ mile/hour}{\underbrace{\dfrac{360°}{24\ hour}}_{\frac{1}{3}}}(z_2^{\circ} - z_1^{\circ})\sec\phi}$

Solution 5 (figure in appendix)

$C = 90° - \phi = NQ + QZ, \cos NQ\cos H = \sin NQ\cot(90°-\delta) - \sin H\cot\ 90° \Rightarrow \tan NQ = \cot\delta\cos H\ \Rightarrow NQ = x$

$\dfrac{\sin H}{\sin QS} = \dfrac{\sin 90°}{\sin(90°-\delta)} \Rightarrow \sin QS = \sin H\cos\delta \Rightarrow QS = y, \cos QZ\cos y + \sin y\sin QZ\cos 90° = \cos z \Rightarrow$

$\cos QZ = \cos z\sec y\ \Rightarrow \boxed{C = \underbrace{x}_{NQ} + \underbrace{\cos^{-1}(\cos z\sec y)}_{QZ}}$

Solution 6 (figure in appendix)

$SS' = \cos^{-1}(\dfrac{-5}{8}), \cos SS' = \sin\delta\sin\delta' + \cos\delta\cos\delta'\cos(\alpha-\alpha') \Rightarrow \alpha - \alpha' = H' - H = 60° = 4^h siderial\ time$

$\xrightarrow{\div 1.002738} \boxed{3^h59^m21^s}\ solar\ mean\ time$

Solution 7

$$\frac{\sin A}{\cos\delta} = \frac{\sin\eta}{\cos\phi} \Rightarrow \sin A = \cos\delta \sec\phi \sin\eta \Rightarrow A_{max}: \eta = 90° \Rightarrow \boxed{A_{max} = \sin^{-1}(\cos\delta \sec\phi)}$$

Solution 8

$$UT \equiv GMT = MT \pm l = GMAT + 12^h m = HAMS + 12^h \pm l, HAMS = 9^h 27^m \coloneqq H, ST = RAMS + HAMS = 9^h 48^m$$

$$\sin\delta = \sin\phi \sin a + \cos a \cos\phi \cos A \Rightarrow \boxed{\delta = 5°.41}, \sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \Rightarrow H = 2^h 15^m 8^s$$

$$ST = H + \alpha \Rightarrow \boxed{\alpha = 7^h 32^m 52^s}, \boxed{Star: Procyon, \alpha\ CMi}$$

Solution 9

$$H = H_{Greenwich} \pm l \Rightarrow H = 19^h 4^m, \sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \Rightarrow \boxed{a = 37°.92}$$

$$\frac{\sin A}{\cos\delta} = \frac{-\sin H}{\cos a} \Rightarrow \boxed{A = 58°.0}$$

Solution 10

$$A_{max} = \sin^{-1}(\cos\delta \sec\phi) \Rightarrow \delta = \cos^{-1}(\cos\phi \sin A_{max}) \Rightarrow \boxed{\delta = 60°}$$

Solution 11

$$\cos z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \xrightarrow{diffrentiating} \sin z\, \Delta z = \cos\phi \cos\delta \sin H\, \Delta H \Rightarrow \Delta H = \frac{\sin z}{\cos\phi \cos\delta \sin H} \Delta z$$

$$\frac{\sin A}{\cos\delta} = \frac{-\sin H}{\sin z} \Rightarrow \Delta H = \frac{-\Delta z}{\sin A \cos\phi} \Rightarrow \boxed{\Delta H = \Delta z \csc A \sec\phi}$$

Solution 12

$$\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \xrightarrow{diffrentiating} \cos a\, \Delta a = \cos\phi \sin\ \delta\, \Delta\phi - \sin\phi \cos\delta \cos H\, \Delta\phi - \cos\phi \cos\delta \sin H\, \Delta H$$

$$\cos a \cos A = \cos\phi \sin\delta - \sin\phi \cos\delta \cos H, \frac{\sin A}{\cos\delta} = \frac{-\sin H}{\cos a} \Rightarrow -\sin H \cos\delta = \sin A \cos a$$

$$\cos a\, \Delta a = \cos a \cos A\, \Delta\,\phi + \cos a \sin A \cos\phi\, \Delta H \Rightarrow \boxed{\Delta a = \Delta\phi \cos A + \Delta H \sin A \cos\phi}$$

Solution 13

$$\cos a \cos A = \cos\phi \sin\delta - \sin\phi \cos\delta \cos H, \sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H\,, H: cte$$

$$\xrightarrow{diffrentiating} \cos a\, \Delta a = (\cos\phi \sin\ \delta\ - \sin\phi \cos\delta \cos H)\, \Delta\phi = \cos a \cos A\ \ \Delta\phi \Rightarrow \Delta\phi = \frac{\Delta a}{\cos A}$$

$$\sin\delta = \sin a \sin\phi + \cos a \cos\phi \cos A \Rightarrow \cos A = \frac{\sin\delta - \sin a \sin\phi}{\cos a \cos\phi} \Rightarrow \boxed{\Delta\phi = \frac{\Delta a \cos a\ cos\ \phi}{\sin\delta - \sin a \sin\phi}}$$

Solution 14

$$\tan\left(\frac{\alpha-\alpha'}{2}-\chi\right)=\frac{\tan\left(\frac{\alpha-\alpha'}{2}\right)-\tan\chi}{1+\tan\left(\frac{\alpha-\alpha'}{2}\right)\tan\chi}=\frac{\sin(\delta'-\delta)}{\sin(\delta'+\delta)}\cot(\frac{\alpha'-\alpha}{2})\coloneqq\beta$$

$$\Rightarrow\tan\chi=\frac{\tan\left(\frac{\alpha-\alpha'}{2}\right)-\beta}{1+\beta\tan(\frac{\alpha-\alpha'}{2})}=\frac{\tan\left(\frac{\alpha-\alpha'}{2}\right)-\frac{\sin(\delta'-\delta)}{\sin(\delta'+\delta)}\cot(\frac{\alpha'-\alpha}{2})}{1+\frac{\sin(\delta'-\delta)}{\sin(\delta'+\delta)}\underbrace{\cot(\frac{\alpha'-\alpha}{2})\tan(\frac{\alpha-\alpha'}{2})}_{-1}}\xrightarrow{simplifying}$$

$$=\frac{\tan\left(\frac{\alpha-\alpha'}{2}\right)\sin\delta'\cos\delta+\tan\left(\frac{\alpha-\alpha'}{2}\right)\cos\delta'\sin\delta-\sin\delta'\cos\delta\cot\left(\frac{\alpha'-\alpha}{2}\right)+\cos\delta'\cos\delta\ \cot\left(\frac{\alpha'-\alpha}{2}\right)}{\sin\delta'\cos\delta+\sin\delta'\cos\delta-\sin\delta'\cos\delta+\cos\delta'\sin\delta}$$

$$=\frac{\sin\delta'\cos\delta\left[\underbrace{\tan\left(\frac{\alpha-\alpha'}{2}\right)+\cot\left(\frac{\alpha-\alpha'}{2}\right)}_{\underbrace{\tan\theta+\cot\theta=\frac{2}{\sin2\theta}}_{\frac{2}{\sin(\alpha-\alpha')}}}\right]+\cos\delta'\sin\delta\left[\underbrace{\tan\left(\frac{\alpha-\alpha'}{2}\right)-\cot\left(\frac{\alpha-\alpha'}{2}\right)}_{\underbrace{\tan\theta-\cot\theta=\frac{-2\cos2\theta}{\sin2\theta}}_{\frac{-2\cos(\alpha-\alpha')}{\sin(\alpha-\alpha')}}}\right]}{2\cos\delta'\sin\delta}=\frac{\tan\delta'\cot\delta-\cos(\alpha-\alpha')}{\sin(\alpha-\alpha')}$$

$\sin\phi\cos H=\cos\phi\tan\delta+\sin H\cot A\,,\ \sin\phi\cos H'=\cos\phi\tan\delta'+\sin H'\cot A',\ A=A',\ H'-H=\alpha-\alpha'$

$$\Rightarrow\ \frac{\sin\phi\cos H-\cos\phi\tan\delta}{\sin H}=\frac{\sin\phi\cos H'-\cos\phi\tan\delta'}{\sin H'}\Rightarrow\tan\delta'=\frac{\sin\phi\sin H\cos H'-\sin\phi\sin H'\cos H+\cos\phi\sin H'\tan\delta}{\sin H\cos\phi}$$

$$=\frac{\sin\phi\sin(H-H')+\cos\phi\tan\delta\sin((H'-H)+H)}{\sin H\cos\phi},\ H-H'=\alpha'-\alpha$$

$$=\frac{-\sin\phi\sin(\alpha-\alpha')+\cos\phi\tan\delta\sin(\alpha-\alpha')\cos H+\cos\phi\tan\delta\cos(\alpha-\alpha')\sin H}{\sin H\cos\phi}$$

$$\tan\chi=\frac{\frac{-\sin\phi\sin(\alpha'-\alpha)+\cos\phi\tan\delta\sin(\alpha-\alpha')\cos H+\cos\phi\tan\delta\cos(\alpha-\alpha')\sin H}{\sin H\cos\phi}\cot\delta-\cos(\alpha-\alpha')}{\sin(\alpha-\alpha')}$$

$$=\frac{-\tan\phi\cot\delta+\cos H}{\sin H}=\frac{\sin\chi}{\cos\chi}\Rightarrow-\tan\phi\cot\delta\cos\chi+\cos H\cos\chi=\sin H\sin\chi\ \Rightarrow\boxed{\cos(\chi+H)=\tan\phi\cos\chi\cot\delta}$$

Solution 15 (figure in appendix)

$x=h\left|\cot a_{\max\,(\delta=0°)}\right|,\ y=h\left|\cot a_{A=90°,\delta=\varepsilon}\right|$

$\sin a=\sin\phi\sin\delta+\cos\phi\cos\delta\cos H\ ,\ \sin\delta=\sin a\sin\phi+\cos a\cos\phi\cos A\ \Rightarrow$

$$\begin{cases}\delta=0°\Rightarrow a_{\max\,(\delta=0°)}=\pm(90°-\phi)\\ \delta=\varepsilon\Rightarrow\sin a_{A=90°,\delta=\varepsilon}=\frac{\sin\varepsilon}{\sin\phi}\Rightarrow a_{A=90°,\delta=\varepsilon}=\psi\end{cases}\Rightarrow x=h\cot\phi\,,\ y=h\cot\psi\ \Rightarrow\boxed{x=y\tan\psi\tan\phi}$$

Solution 16 (figure in appendix)

$\delta = 0°, A = 180° + \theta, \sin\phi\cos H = \cos\phi\tan\delta + \sin H\cot A \Rightarrow \boxed{\tan H = \sin\phi\tan\theta}$

$\sin a = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \overset{\delta=H=0°}{\Rightarrow} \delta = a_{\max\,(\delta=0°)} = \pm(90° - \phi), b = h\cot a_{\max\,(\delta=0°)}\sin\theta = \boxed{h\tan\phi\sin\theta}$

Solution 17 (figure in appendix)

$A: west = 90° \Rightarrow \delta = 0°, \alpha = \tan^{-1}(\frac{91\,cm\,.\tan 30°}{1.6\,km}) = 0°.019, \frac{\sin x}{\sin 120°} = \frac{\sin\alpha}{\sin 10°} \Rightarrow x = 0°.094\,, \Delta T = \frac{x}{360°} \times 24^h \cong \boxed{22.5\,s}$

Solution 18

$\cos\delta\cos\frac{\Delta L}{2} = \sin\delta\cot(90° - \phi) - \sin\frac{\Delta L}{2}\cot 90° \Rightarrow \tan\delta\tan\phi = \cos\frac{\Delta L}{2}$

$\cos H_{a=0°} = -\tan\delta\tan\phi \;\Rightarrow NT = 2 \times (180° - \cos^{-1}(\underbrace{-\tan\delta\tan\phi}_{-\cos\frac{\Delta L}{2}})) = 2\frac{\Delta L}{2} \Rightarrow \boxed{NT = \Delta L}$

Solution 19 (figure in appendix)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\delta\cos(\alpha-\theta) \\ \cos\delta\sin(\alpha-\theta) \\ \sin\delta \end{bmatrix}, \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\delta'\cos(\alpha'-\theta') \\ \cos\delta'\sin(\alpha'-\theta') \\ \sin\delta' \end{bmatrix}, \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \begin{cases} x' = x \Rightarrow & \boxed{\cos\delta'\cos(\alpha'-\theta') = \cos\delta\cos(\alpha-\theta)} \\ y' = y\cos i + z\sin i \Rightarrow & \boxed{\cos\delta'\sin(\alpha'-\theta') = \cos\delta\sin(\alpha-\theta)\cos i + \sin\delta\sin i} \\ z' = -y\sin i + z\cos i \Rightarrow & \boxed{\sin\delta' = -\cos\delta\sin(\alpha'-\theta')\sin i + \sin\delta\cos i} \end{cases}$$

$$\begin{matrix} \cos\delta'\cos(\alpha'-\theta') = -0.740 \\ \cos\delta'\sin(\alpha'-\theta') = -0.466 \\ \sin\delta' = 0.485 \end{matrix}, \begin{cases} \sin\delta' = 0.485 \Rightarrow & \delta' = 29°0' \\ \cos\delta'\cos(\alpha'-\theta') = -0.740 \Rightarrow & \alpha' = 262°47'\,or\,327°12' \\ \cos\delta'\sin(\alpha'-\theta') = -0.466 \Rightarrow & \alpha' = 327°12'\,or\,82°48' \end{cases} \Rightarrow \boxed{\alpha' = 327°12'}\,, \quad \boxed{\delta' = 29°0'}$$

Solution 20

$Solution\ 5 \Rightarrow 90° - \phi = \tan^{-1}(\tan p\cos H) + \cos^{-1}(\frac{\sin a}{\cos y}), \sin y = \sin p\sin H \Rightarrow \phi = 90° - \tan^{-1}(\tan p\cos H) - \cos^{-1}(\frac{\sin a}{\cos y})$

$\phi = \phi_{p=0} + p(\frac{\partial\phi}{\partial p})_{p=0} + \frac{p^2}{2}(\frac{\partial^2\phi}{\partial p^2})_{p=0} + \cdots, \phi_{p=0} = 90 - \tan^{-1}(\tan 0°\cos H) - \cos^{-1}(\frac{\sin a}{\cos 0°}) = a$

$$\left(\frac{\partial\phi}{\partial p}\right)_{p=0} = \left(-\frac{\frac{1}{\cos^2 p}\cos H}{1 + \tan^2 p\cos^2 H} + \frac{\frac{-\sin y\sin a}{\cos^2 y}\frac{\partial y}{\partial p}}{\sqrt{1 - \frac{\sin^2 a}{\cos^2 y}}}\right)_{p=0} = -\cos H$$

$$\left(\frac{\partial^2\phi}{\partial p}\right)_{p=0} = (-\frac{\left(\frac{2\sin p}{\cos^3 p}\cos H\right)(1 + \tan^2 p\cos^2 H) - (\frac{1}{\cos^2 p}\cos H)(\frac{2\cos^2 H\sin p}{\cos^3 p})}{(1 + \tan^2 p\cos^2 H)^2}$$

$$+\frac{\sqrt{1-\frac{\sin^2 a}{\cos^2 y}}\left[\frac{-\cos^3 y-2\cos y\sin^2 y}{\cos^4 y}\sin a\left(\frac{\partial y}{\partial p}\right)^2+\frac{-\sin y\sin a}{\cos^2 y}\frac{\partial^2 y}{\partial p^2}\right]-\frac{\left(-2\sin^2 a\,\frac{\sin y}{\cos^3 y}\right)}{2\sqrt{1-\frac{\sin^2 a}{\cos^2 y}}}\left(\frac{-\sin y\sin a}{\cos^2 y}\frac{\partial y}{\partial p}\right)}{1-\frac{\sin^2 a}{\cos^2 y}}\Bigg)_{p=0}$$

$$(\sin y)_{p=0}=0\,,(\cos y)_{p=0}=1\,,\left(\frac{\partial y}{\partial p}\right)_{p=0}=\sin H\ ,\left(\frac{\partial^2 y}{\partial p^2}\right)_{p=0}=0\ \Rightarrow\left(\frac{\partial^2\phi}{\partial p^2}\right)_{p=0}=0+\frac{\pm\cos a\,[-\sin a\sin^2 H]-0}{\cos^2 a}=\mp\tan a\sin^2 H$$

$$\xrightarrow{accepting+}\ \phi=\phi_{p=0}+p\left(\frac{\partial\phi}{\partial p}\right)_{p=0}+\frac{p^2}{2}\left(\frac{\partial^2\phi}{\partial p^2}\right)_{p=0}+\cdots=a-p\cos H+\frac{p^2}{2}\tan a\sin^2 H+\cdots\xrightarrow{radians\ to\ arc\ seconds}$$

$$\phi\sin 1''=a\sin 1''-p\sin 1''\cos H+\frac{(p\sin 1'')^2}{2}\tan a\sin^2 H+\cdots\Rightarrow\boxed{\phi\cong a-p\cos H+\frac{p^2}{2}\tan a\sin^2 H\sin 1''}$$

Solution 21

$$\cos z=\sin\phi\sin\delta+\cos\phi\cos\delta\cos H\ ,z=z_{H=0}+H\left(\frac{dz}{dH}\right)_{H=0}+\frac{H^2}{2}\left(\frac{d^2z}{dH^2}\right)_{H=0}+\frac{H^3}{6}\left(\frac{d^3z}{dH^3}\right)_{H=0}+\frac{H^4}{24}\left(\frac{d^4z}{dH^4}\right)_{H=0}+\cdots,z_{H=0}=\phi-\delta$$

$$\xrightarrow{Derivating\ main\ formula}-\sin z\frac{dz}{dH}=-\cos\phi\cos\delta\sin H\Rightarrow\left(\frac{dz}{dH}\right)_{H=0}=0$$

$$\xrightarrow{Derivating\ formula\ above}-\cos z\left(\frac{dz}{dH}\right)^2-\sin z\frac{d^2z}{dH^2}=-\cos\phi\cos\delta\cos H\Rightarrow\left(\frac{d^2z}{dH^2}\right)_{H=0}=\frac{\cos\phi\cos\delta}{\sin z_{H=0}}=\frac{\cos\phi\cos\delta}{\sin(\phi-\delta)}=$$

$$\xrightarrow{Derivating\ formula\ above}\sin z\left(\frac{dz}{dH}\right)^3\underbrace{-2\cos z\frac{dz}{dH}\frac{d^2z}{dH^2}-\cos z\frac{dz}{dH}\frac{d^2z}{dH^2}}_{-3\cos z\frac{dz}{dH}\frac{d^2z}{dH^2}}-\sin z\frac{d^3z}{dH^3}=\cos\phi\cos\delta\sin H\Rightarrow\left(\frac{d^3z}{dH^3}\right)_{H=0}=0$$

$$\xrightarrow{Derivating\ formula\ above}\cos z\left(\frac{dz}{dH}\right)^4+3\sin z\left(\frac{dz}{dH}\right)^2\frac{d^2z}{dH^2}+3\sin z\left(\frac{dz}{dH}\right)^2\frac{d^2z}{dH^2}-3\cos z\left(\frac{d^2z}{dH^2}\right)^2-3\cos z\frac{dz}{dH}\frac{d^3z}{dH^3}-\cos z\frac{dz}{dH}\frac{d^3z}{dH^3}$$

$$-\sin z\frac{d^4z}{dH^4}=\cos\phi\cos\delta\cos H\Rightarrow\left(\frac{d^4z}{dH^4}\right)_{H=0}=-\frac{\cos\phi\cos\delta+3\cos z_{H=0}\left(\frac{d^2z}{dH^2}\right)_{H=0}}{\sin z_{H=0}}=-\frac{\cos\phi\cos\delta}{\sin(\phi-\delta)}-3\cot(\phi-\delta)\frac{\cos^2\phi\cos^2\delta}{\sin^2(\phi-\delta)}$$

$$\Rightarrow\left(\frac{d^2z}{dH^2}\right)_{H=0}=\frac{\alpha_1}{2\sin^2\frac{H}{2}}\cong\frac{2\,\alpha_1}{H^2}\,,\left(\frac{d^4z}{dH^4}\right)_{H=0}\cong-\frac{2\,\alpha_1}{H^2}-12\cot(\phi-\delta)\frac{{\alpha_1}^2}{H^4}=-\frac{2\,\alpha_1}{H^2}-24\,\alpha_2$$

$$\Rightarrow z\cong\ \phi-\delta+H\,(0)+\frac{H^2}{2}\frac{2\,\alpha_1}{H^2}+\frac{H^3}{6}(0)+\frac{H^4}{24}\left(-\frac{2\,\alpha_1}{H^2}-\frac{2\,\alpha_1}{H^2}-24\,\alpha_2\right)\cong\ \phi-\delta+\left(\underbrace{1-\frac{H^2}{12}}_{\cong 1}\right)\alpha_1-\ \alpha_2\Rightarrow\boxed{z\cong\phi-\delta+\alpha_1-\alpha_2}$$

Solution 22

$$\sin\delta=\sin a\sin\phi+\cos a\cos\phi\cos A\,,\sin\delta=\sin\varepsilon\sin L\ \overset{A=90^\circ}{\Rightarrow}\ \sin\varepsilon\sin L=\sin a\sin\phi\ \Rightarrow\boxed{\phi=\sin^{-1}(\sin L\sin\varepsilon\csc a)}$$

Solution 23

$$sin\,\phi\cos H = \cos\phi\tan\delta + \sin H\cot A \overset{\phi=45°,A=90°}{\Rightarrow} \cos H_1 = \tan\delta\,, 24^h > H_1 > 12^h \Rightarrow \sin H_1 = -\sqrt{1-\tan^2\delta}$$

$$\cos H_2 = -\tan\phi\tan\delta \overset{\phi=45°}{\Rightarrow} \cos H_2 = -\tan\delta\,, 0^h < H_2 < 12^h \Rightarrow \sin H_1 = \sqrt{1-\tan^2\delta}$$

$$\cos(H_2 - H_1) = \cos H_2\cos H_1 + \sin H_2\sin H_1 = -\tan^2\delta - 1 + \tan^2\delta \Rightarrow \cos(H_2 - H_1) = -1 \Rightarrow \boxed{H_2 - H_1 = 12^h: cte}$$

Solution 24

$$Solution\ 7 \Rightarrow A_{max}: \eta = 90°\,, \cos(90° - a)\cos\eta = \sin(90° - a)\cot(90° - \delta) - \sin\eta\cot A \overset{\eta=90°}{\Rightarrow} \tan A_{max} = \frac{1}{\tan\delta\cos a}$$

$$\frac{\sin A}{\cos\delta} = \frac{\sin\eta}{\cos\phi} = \frac{-\sin H}{\cos a} \Rightarrow \sin H = \frac{-\cos a\sin A_{max}}{\cos\delta}\,, \sin A_{max} = \frac{\cos\delta}{\cos\phi}$$

$$A = A_{max} + (H - H_{A_{max}})\underbrace{(\frac{dA}{dH})_{A_{max}}}_{0} + \frac{\left(\overbrace{H - H_{A_{max}}}^{t}\right)^2}{2}(\frac{d^2A}{dH^2})_{A_{max}} + \cdots \Rightarrow \Delta A \cong \frac{t^2}{2}(\frac{d^2A}{dH^2})_{A_{max}}$$

$$sin\,\phi\cos H = \cos\phi\tan\delta + \sin H\cot A \xrightarrow{derivation} -\sin\phi\sin H = \cos H\cot A - \frac{\sin H}{\sin^2 A}\frac{dA}{dH} \xrightarrow{derivation}$$

$$-\sin\phi\cos H = -\sin H\cot A - \frac{\sin H}{\sin^2 A}\frac{dA}{dH} - \frac{\cos H}{\sin^2 A}\frac{dA}{dH} + \frac{2\sin H\cos A}{\sin^3 A}\frac{dA}{dH} - \frac{\sin H}{\sin^2 A}\frac{d^2A}{dH^2} \overset{(\frac{dA}{dH})_{A_{max}}=0}{\Rightarrow} \sin\phi\cos H - \sin H\cot A_{max} =$$

$$\cos\phi\tan\delta = \frac{\sin H}{\sin^2 A_{max}}(\frac{d^2A}{dH^2})_{A_{max}} \Rightarrow (\frac{d^2A}{dH^2})_{A_{max}} = \frac{\cos\phi\tan\delta\ \sin^2 A_{max}}{\sin H} \overset{\text{جاگذاری}}{=} -\sin^2\delta\tan A_{max} \Rightarrow \Delta A \cong -\frac{t^2}{2}\sin^2\delta\tan A_{max}$$

$$\xrightarrow{radians\ to\ seconds\ and\ arc\ seconds} \Delta A\sin 1'' \cong -\frac{(15\,t\sin 1'')^2}{2}\sin^2\delta\tan A_{max} \Rightarrow \boxed{\Delta A \cong -\frac{15^2t^2}{2}\sin 1''\sin^2\delta\tan A_{max}}$$

Solution 25

$$\frac{\sin A}{\cos\delta} = \frac{\sin\eta}{\cos\phi} = \frac{-\sin H}{\sin z}\,, \cos\phi\cos A = \sin\delta\cos z - \cos\delta\sin z\cos\eta\,, solution\ 21 \Rightarrow \frac{dz}{dH} = -\sin A\cos\phi = -\sin\eta\cos\delta$$

$$\sin z\cos\eta = \sin\phi\cos\delta - \cos\phi\sin\delta\cos H \xrightarrow{derivation} -\cos z\cos\eta\overbrace{\sin\eta\cos\delta}^{-\frac{dz}{dH}} - \sin z\sin\eta\frac{d\eta}{dH} = -\cos\phi\sin\delta\sin H = \sin z\sin\eta\sin\delta$$

$$\Rightarrow \frac{d\eta}{dH} = \frac{-(\cos z\cos\eta\cos\delta - \sin z\sin\delta)}{\sin z} \Rightarrow \boxed{\frac{d\eta}{dH} = \cos\phi\cos A\csc z}\,, \frac{d^2z}{dH^2} = \frac{d}{dz}(\frac{dz}{dH}) = \boxed{-\cos\eta\cos\delta\,\frac{d\eta}{dH}}$$

$$\sin A\cos\phi = \sin\eta\cos\delta \xrightarrow{derivation} \cos A\cos\phi\frac{dA}{dH} = \cos\delta\cos\eta\frac{d\eta}{dH} = \cos\delta\cos\eta\cos\phi\cos A\ \csc z = \frac{\cos\delta\cos\eta\ \cos\phi\cos A}{\sin z}$$

$$\Rightarrow \frac{dA}{dH} = \frac{\cos\delta\cos\eta}{\sin z} \xrightarrow{derivation} \boxed{\frac{d^2A}{dH^2} = \cos\delta\,\frac{-\sin\eta\sin z\frac{d\eta}{dH} + \cos\eta\cos z\frac{dz}{dH}}{\sin^2 z}}$$

Solution 26

$$\cos H = -\tan\phi\tan\delta = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} \Rightarrow 1-\cos H = 2\sin^2\frac{H}{2} = 1+\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = \frac{\cos\phi\cos\delta+\sin\phi\sin\delta}{\cos\phi\cos\delta} = \frac{\cos(\phi-\delta)}{\cos\phi\cos\delta}$$

$$1+\cos H = 2\cos^2\frac{H}{2} = 1-\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = \frac{\cos\phi\cos\delta-\sin\phi\sin\delta}{\cos\phi\cos\delta} = \frac{\cos(\phi+\delta)}{\cos\phi\cos\delta} \Rightarrow \boxed{\tan^2\frac{H}{2} = \frac{\cos(\phi-\delta)}{\cos(\phi+\delta)}}$$

Solution 27

$$\cos H_1 = -\tan\phi\tan\delta_1 \text{ , } \cos H = -\tan\phi\tan\delta \text{ , } 2H_1 = H \text{ , } \cos(2H) = -\tan\phi\tan\delta = 2\cos^2 H - 1 = 2\tan^2\phi\tan^2\delta_1 - 1$$

$$\Rightarrow \boxed{\tan\phi\tan\delta = 1-2\tan^2\phi\tan^2\delta_1}$$

Solution 28

$$\cos H = -\tan\phi\tan\delta \Rightarrow \sin H = -\sqrt{1-\tan^2\phi\tan^2\delta} \text{ , } \cos H_1 = -\tan\phi\tan\delta_1 \Rightarrow \sin H_1 = -\sqrt{1-\tan^2\phi\tan^2\delta_1}$$

$$\cos(H-H_1) = \cos H\cos H_1 + \sin H\sin H_1 = \tan^2\phi\tan\delta\tan\delta_1 + \sqrt{1+\tan^4\phi\tan^2\delta\tan^2\delta_1 - \tan^2\phi\tan^2\delta - \tan^2\phi\tan^2\delta_1} \xrightarrow{sqr}$$

$$1-\sin^2(H-H_1) = 2\tan^4\phi\tan^2\delta\tan^2\delta_1 + 1 - \tan^2\phi\tan^2\delta - \tan^2\phi\tan^2\delta_1 +$$

$$2\tan^2\phi\tan^2\delta\tan^2\delta_1\sqrt{1+\tan^4\phi\tan^2\delta\tan^2\delta_1 - \tan^2\phi\tan^2\delta - \tan^2\phi\tan^2\delta_1}$$

$$\Rightarrow 2\tan^2\phi\tan^2\delta\tan^2\delta_1\cos(H-H_1) - 1 + \sin^2(H-H_1) = -1+\tan^2\phi\tan^2\delta + \tan^2\phi\tan^2\delta_1 \text{ , } H-H_1 = \alpha_1-\alpha \overset{\times\cot^2\phi}{\Rightarrow}$$

$$2\tan^2\delta\tan^2\delta_1\cos(\alpha_1-\alpha) - \cot^2\phi + cot^2\phi\sin^2(\alpha_1-\alpha) = -\cot^2\phi + \tan^2\delta + \tan^2\delta_1$$

$$\Rightarrow \boxed{\cot^2\phi\sin^2(\alpha_1-\alpha) = \tan^2\delta + \tan^2\delta_1 - 2\tan^2\delta\tan^2\delta_1\cos(\alpha_1-\alpha)}$$

Solution 29

$$\cos A_{a=0^\circ} = \frac{\sin\delta}{\cos\phi}, \cos H_{a=0^\circ} = -\tan\phi\tan\delta \xrightarrow{differentiating} -\sin A_{a=0^\circ}\Delta A_{a=0^\circ} = \frac{\cos\delta\,\Delta\delta}{\cos\phi}, -\sin H_{a=0^\circ}\underbrace{\Delta H_{a=0^\circ}}_{m} = \frac{-\sin\phi}{\cos\phi\cos^2\delta}\Delta\delta$$

$$\frac{\sin A}{\cos\delta} = \frac{-\sin H}{\cos a} \Rightarrow \sin A_{a=0^\circ} = -\sin H_{a=0^\circ}\cos\delta \Rightarrow m = \frac{\sin\phi}{\cos\phi\cos^2\delta}\frac{\cos\phi\sin A_{a=0^\circ}}{\cos\delta}\Delta A_{a=0^\circ} \overset{rad\ to\ mins}{\Rightarrow} \boxed{\Delta A_{a=0^\circ} = 15m\cos^2\delta\csc\phi}$$

Solution 30

$$\sin a = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \overset{Equator}{\Rightarrow} \sin(-18^\circ) = \cos\delta\cos H \Rightarrow H = \cos^{-1}(-\sin 18^\circ\sec\delta)$$

$$H-\frac{\pi}{2} = \sin^{-1}(\sin 18^\circ\sec\delta), \Delta T = 2\left(H-\frac{\pi}{2}\right) = 2\ \sin^{-1}(\sin 18^\circ\sec\delta) \overset{radians\ to\ hours}{\Rightarrow} \boxed{\Delta T = \frac{12}{\pi}\sin^{-1}(\sin 18^\circ\sec\delta)}$$

Solution 31

$$solution\ 35 \Rightarrow 2\cos^2\phi\sin^2\frac{T}{2} = 1-\cos\theta\cos(\eta'-\eta) \Rightarrow T = f(\eta'-\eta) \Rightarrow \left(\frac{dT}{d(\eta'-\eta)}\right)_{T_{max}} = 0$$

$$\xrightarrow{derivation} 2\cos^2\phi\sin\frac{T}{2}\cos\frac{T}{2}\frac{dT}{d(\eta'-\eta)} = \cos\theta\sin(\eta'-\eta) \overset{\left(\frac{dT}{d(\eta'-\eta)}\right)_{T_{min}}=0}{\Rightarrow} sin(\eta'-\eta)_{T_{max}} = 0 \Rightarrow \cos(\eta'-\eta)_{T_{max}} = 1$$

$$2\cos^2\phi\sin^2\frac{T_{max}}{2} = 1-\cos\theta = 2\sin^2\frac{\theta}{2} \Rightarrow T_{max} = 2\sin^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\phi}\right) \overset{\theta=18^\circ,\,radians\ to\ hours}{\Rightarrow} \boxed{T_{min} = \frac{2}{15}\sin^{-1}(\sin 9^\circ \sec\phi)}$$

Solution 32

$$\sin a = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \Rightarrow \cos H = \frac{-\sin 18^\circ - \sin\phi\sin\delta}{\underbrace{\cos\phi\cos\delta}_{>0}}, -\sin 18^\circ - \sin\phi\sin\delta < 0 \Leftrightarrow |\delta| < 18^\circ$$

$$\Rightarrow \cos H < 0 \Rightarrow H > 90^\circ, T = 2H \Rightarrow T > 180^\circ\ or\ \boxed{T > 12^h}$$

Solution 33

$$\sin a = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \Rightarrow \begin{cases} \cos H = \dfrac{-\sin\theta - \sin\phi\sin(-\varepsilon+\chi)}{\cos\phi\cos(-\varepsilon+\chi)} \\ \cos H' = \dfrac{-\sin\theta - \sin\phi\sin(-\varepsilon)}{\cos\phi\cos(-\varepsilon)} \end{cases}, \sin\chi \cong \chi, \cos\chi \cong 1, H < H' \Rightarrow \cos H > \cos H'$$

$$\frac{-\sin\theta - \sin\phi\sin(-\varepsilon+\chi)}{\cos\phi\cos(-\varepsilon+\chi)} > \frac{-\sin\theta - \sin\phi\sin(-\varepsilon)}{\cos\phi\cos(-\varepsilon)} \Rightarrow \frac{-\sin\theta + \sin\phi\sin\varepsilon - \chi\sin\phi\cos\varepsilon}{\cos\varepsilon + \chi\sin\varepsilon} > \frac{-\sin\theta + \sin\phi\sin\varepsilon}{\cos\varepsilon} \Rightarrow$$

$$-\sin\theta\cos\varepsilon + \sin\phi\sin\varepsilon\cos\varepsilon - \chi\sin\phi\cos^2\varepsilon > -\sin\theta\cos\varepsilon - \chi\sin\theta\sin\varepsilon + \sin\phi\, sin\,\varepsilon\cos\varepsilon + \chi\sin\phi\sin^2\varepsilon \Rightarrow$$

$$\chi\sin\theta\sin\varepsilon > \chi\sin\phi \Rightarrow \boxed{\sin\theta\sin\varepsilon > \sin\phi}$$

Solution 34

$$\phi > 90^\circ - \delta - 18^\circ \Rightarrow \delta > 90^\circ - (\phi + 18^\circ), \sin\delta = \sin\varepsilon\sin\lambda, \lambda = \frac{360^\circ}{365}d$$

$$\lambda_s = \sin^{-1}\left(\frac{\cos(\phi+18^\circ)}{\sin\varepsilon}\right) \Rightarrow T = \left[2\frac{365}{360^\circ}(90-\lambda_s)\right] = \boxed{\left[\frac{73}{36}\cos^{-1}\left(\frac{\cos(\phi+18^\circ)}{\sin\varepsilon}\right)\right]}$$

Solution 35

$$\sin a = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H, \cos a\cos\eta = \sin\phi\cos\delta - \cos\phi\sin\delta\cos H, \frac{\sin\eta}{\cos\phi} = \frac{-\sin H}{\cos a}$$

$$\begin{cases} a = 0 \Rightarrow \cos H' = \dfrac{-\sin\phi\sin\delta}{\cos\phi\cos\delta}, \cos\eta' = \sin\phi\cos\delta - \cos\phi\sin\delta\cos H', \sin\eta' = -\sin H'\cos\phi \\ a = -\theta \Rightarrow \cos H = \dfrac{-\sin\theta - \sin\phi\sin\delta}{\cos\phi\cos\delta}, \cos\eta = \dfrac{\sin\phi\cos\delta - \cos\phi\sin\delta\cos H}{\cos\theta}, \sin\eta = \dfrac{-\sin H\cos\phi}{\cos\theta} \end{cases}, H - H' = T$$

$$\cos(\eta' - \eta) = \cos\eta\cos\eta' + \sin\eta\sin\eta' =$$

$$\left(\frac{\sin\phi\cos\delta - \cos\phi\sin\delta\cos H}{\cos\theta}\right)(\sin\phi\cos\delta - \cos\phi\sin\delta\cos H') + \left(\frac{-\sin H\cos\phi}{\cos\theta}\right)(-\sin H'\cos\phi) =$$

$$\frac{1}{\cos\theta}[\sin^2\phi\cos^2\delta - \sin\phi\cos\phi\sin\delta\cos\delta\,(\cos H + \cos H') + \sin^2\delta\cos^2\phi\cos H\cos H' + \cos^2\phi\sin H'\sin H] =$$

$$\frac{1}{\cos\theta}[\sin^2\phi\cos^2\delta + \sin\phi\sin\delta\sin\theta + 2\sin^2\phi\sin^2\delta - \cos^2\delta\cos^2\phi\cos H\cos H' + \cos^2\phi\cos(H - H')] =$$

$$\frac{1}{\cos\theta}[\sin^2\phi\cos^2\delta+\sin\phi\sin\delta\sin\theta+2\sin^2\phi\sin^2\delta-\sin\phi\sin\delta\sin\theta-\sin^2\phi\sin^2\delta+\cos^2\phi\cos(H-H')]=$$

$$\frac{1}{\cos\theta}[\sin^2\phi+\cos^2\phi\cos(H-H')]=\frac{1}{\cos\theta}[1-\cos^2\phi\,(1-\cos(H-H'))]=\frac{1}{\cos\theta}\left(1-2\cos^2\phi\sin^2\left(\frac{H-H'}{2}\right)\right)\Rightarrow$$

$$\cos\theta\cos(\eta'-\eta)=1-2\cos^2\phi\sin^2\frac{T}{2}\Rightarrow\boxed{2\cos^2\phi\sin^2\frac{T}{2}=1-\cos\theta\cos(\eta'-\eta)}$$

Solution 36

$$\begin{cases}\sin\beta=\sin\delta\cos\varepsilon-\sin\varepsilon\cos\delta\sin\alpha\\ \cos\beta\cos\lambda=\cos\alpha\cos\delta\\ \cos\beta\sin\lambda=\sin\delta\sin\varepsilon+\cos\delta\cos\varepsilon\sin\alpha\end{cases}\Rightarrow\begin{cases}\sin\beta=-0.276\ \Rightarrow\quad\beta=-16°2'\\ \cos\beta\cos\lambda=0.048\ \Rightarrow\quad\lambda=87°10'\,or\ 272°\,50'\\ \cos\beta\sin\lambda=0.960\ \Rightarrow\quad\lambda=87°10'\,or\ 92°50'\end{cases}\Rightarrow\boxed{\beta=-16°2'},\boxed{\lambda=87°10'}$$

Solution 37

$$\cos\beta\cos\lambda=\cos\alpha\cos\delta\ \Rightarrow\cos\beta=\frac{\cos\alpha\cos\delta}{\cos\lambda},\cos\beta\sin\lambda=\sin\delta\sin\varepsilon+\cos\delta\cos\varepsilon\sin\alpha\Rightarrow\tan\lambda=\frac{\sin\delta\sin\varepsilon+\cos\delta\cos\varepsilon\sin\alpha}{\cos\alpha\cos\delta}$$

$$\tan\lambda_1=\tan\lambda_2\Rightarrow\frac{\sin\delta_1\sin\varepsilon+\cos\delta_1\cos\varepsilon\sin\alpha_1}{\cos\alpha_1\cos\delta_1}=\frac{\sin\delta_2\sin\varepsilon+\cos\delta_2\cos\varepsilon\sin\alpha_2}{\cos\alpha_2\cos\delta_2}\Rightarrow$$

$$\sin\delta_1\sin\varepsilon\ \cos\alpha_2\cos\delta_2+\ \cos\delta_1\cos\varepsilon\sin\alpha_1\cos\alpha_2\cos\delta_2=\sin\delta_2\sin\varepsilon\ \cos\alpha_1\cos\delta_1+\ \cos\delta_2\cos\varepsilon\sin\alpha_2\cos\alpha_1\cos\delta_1\Rightarrow$$

$$\cos\delta_1\cos\delta_2\cos\varepsilon\sin(\alpha_1-\alpha_2)=\sin\varepsilon\,(\cos\alpha_1\cos\delta_1\sin\delta_2-\cos\alpha_2\cos\delta_2\sin\delta_1)$$

$$\Rightarrow\boxed{\sin(\alpha_1-\alpha_2)=\tan\varepsilon\,(\cos\alpha_1\tan\delta_2-\cos\alpha_1\tan\delta_2)}$$

Solution 38

$$\cos\beta\cos\lambda=\cos\alpha\cos\delta\,,\alpha:cte\xrightarrow{differentiating}-\sin\beta\cos\lambda\,\Delta\beta-\cos\beta\sin\lambda\,\Delta\lambda=-\cos\alpha\sin\delta\,\Delta\delta\overset{\beta=0}{\Rightarrow}\Delta\lambda=\frac{\cos\alpha\sin\delta\,\Delta\delta}{\sin\lambda}$$

$$\frac{\sin\psi}{\sin\alpha}=\frac{\sin\varepsilon}{\sin\delta},\frac{\sin\psi}{\sin\beta}=\frac{\sin 90°}{\sin\Delta\delta}\ \Rightarrow\Delta\delta=\frac{\beta}{\sin\psi}=\frac{\beta\ \overbrace{\sin\delta}^{\cong\sin\lambda\sin\varepsilon}}{\sin\alpha\sin\varepsilon}\Rightarrow\Delta\lambda=\beta\frac{\cos\alpha\sin\delta\sin\lambda\sin\varepsilon}{\sin\lambda\sin\alpha\sin\varepsilon}=\boxed{\beta\sin\delta\cot\alpha}$$

Solution 39

$$\tan\varepsilon=\frac{\tan\delta}{\sin(\frac{\pi}{2}--2q)}=\frac{\tan\delta}{\cos 2q}\cong\frac{\tan\delta}{1-\frac{(2q)^2}{2}}\cong\tan\delta\,(1+2q^2)\Rightarrow\varepsilon\cong\tan^{-1}(\tan\delta+2q^2\tan\delta)$$

$$\tan^{-1}(x+\Delta x)-\tan^{-1}x\cong\frac{\Delta x}{1+x^2}\Rightarrow\tan^{-1}(\tan\delta+2q^2\tan\delta)-\tan^{-1}(\tan\delta)\cong\frac{2q^2\tan\delta}{1+\tan\delta^2}=\frac{\frac{2q^2\sin\delta}{\cos\delta}}{\frac{1}{\cos^2\delta}}=q^2\sin 2\delta$$

$$\Rightarrow\ \varepsilon-\delta\cong q^2\sin 2\delta\Rightarrow\boxed{\varepsilon\cong\delta+q^2\sin 2\delta}$$

Solution 40

$$\cos GS=\sin\delta_G\sin\delta_s+\cos\delta_G\cos\delta_G\cos(\alpha_G-\alpha_s)=\cos 90°=0\Rightarrow\cos(\alpha_G-\alpha_s)=-\tan\delta_G\tan\delta_s\,,\ \tan\delta_s=\tan\varepsilon\sin\alpha_s$$

$\Rightarrow \cos\alpha_G \cos\alpha_s + \sin\alpha_G \sin\alpha_s = -\tan\delta_G \tan\varepsilon \sin\alpha_s \Rightarrow \tan\alpha_s = -\dfrac{\tan\delta_G \tan\varepsilon + \sin\alpha_G}{\cos\alpha_G}$, $\tan\alpha_s = \tan\lambda_s \cos\varepsilon$

$$\Rightarrow \tan\lambda_s = -\frac{\tan\delta_G \tan\varepsilon + \sin\alpha_G}{\cos\alpha_G \cos\varepsilon} \Rightarrow \begin{cases} \lambda_s = 0°.84 \Rightarrow \boxed{22\ march} \\ \lambda_s = 180°.84 \Rightarrow \boxed{\text{2 مهر}} \end{cases}$$

Solution 41 (figure in appendix)

$$\begin{cases} \sin\theta\, dr = \cos\delta\, d\alpha \\ \cos\theta\, dr = -d\delta \end{cases}, \frac{\sin\theta}{\cos\delta_0} = \frac{\sin(\alpha-\alpha_0)}{\sin r} \Rightarrow \sin\theta = \cos\delta_0 \sin(\alpha-\alpha_0)\csc r \Rightarrow \boxed{\cos\delta\, d\alpha = \cos\delta_0 \sin(\alpha-\alpha_0)\csc r\ dr}$$

$$\sin r\cos\theta = \sin\delta_0\cos\delta - \cos\delta_0\sin\delta\cos(\alpha-\alpha_0) \Rightarrow \boxed{d\delta = [\cos\delta_0\sin\delta\cos(\alpha-\alpha_0) - \sin\delta_0\cos\delta]\csc r\, dr}$$

Solution 42

$$T = \alpha_k + H_k\ , \alpha_k = 18^h, \delta_k = \frac{\pi}{2} - \varepsilon, \cos z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H, H = T - 18^h$$

$$\Rightarrow \boxed{z_k = \cos^{-1}(\sin\phi\cos\varepsilon - \sin\varepsilon\cos\phi\sin T)}$$

Notes:

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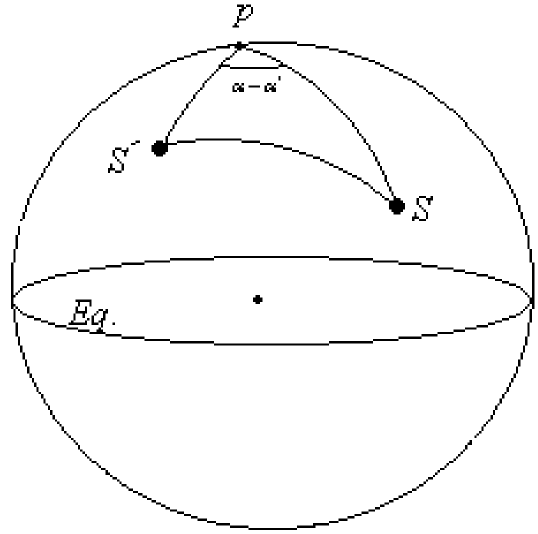
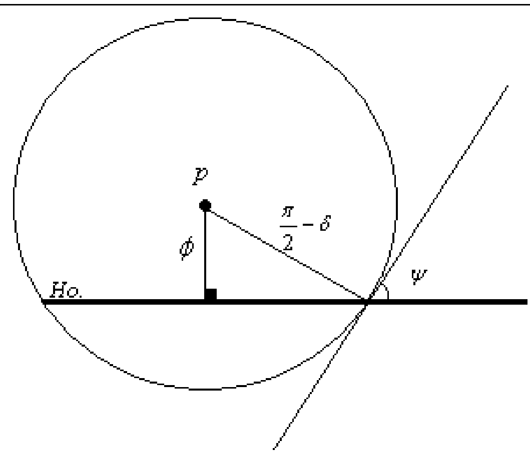
The End.

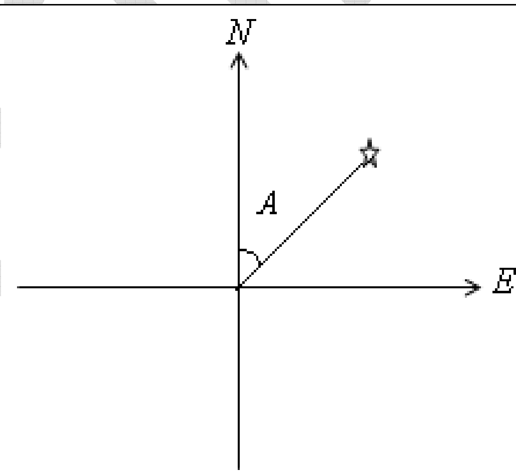
Appendix



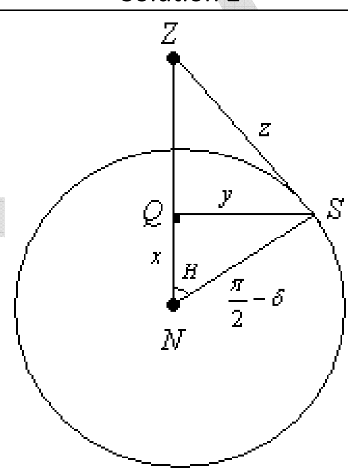
Base Figure



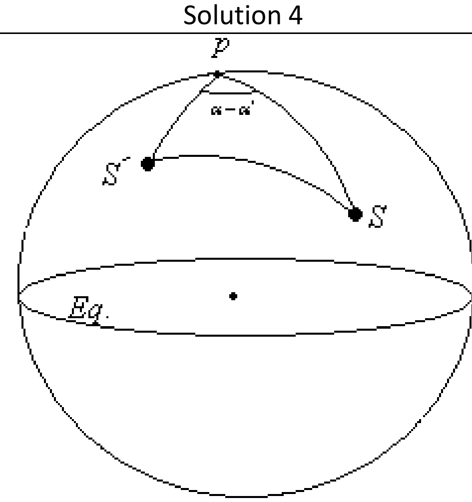
Solution 2



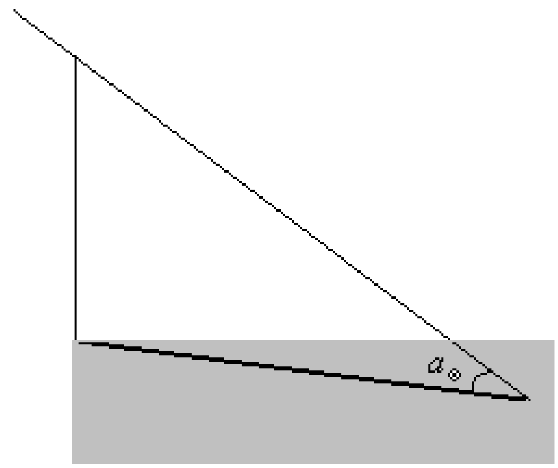
Solution 4



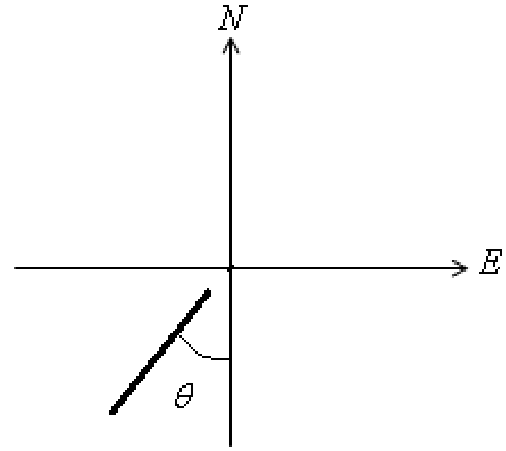
Solution 5



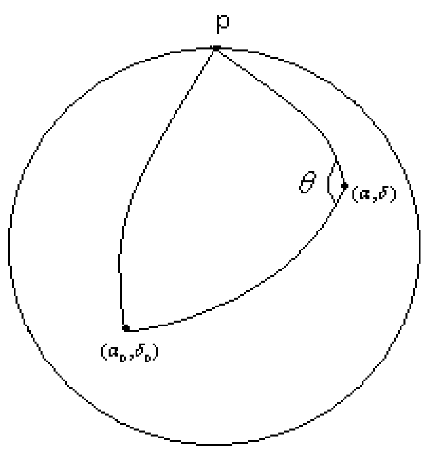
Solution 6



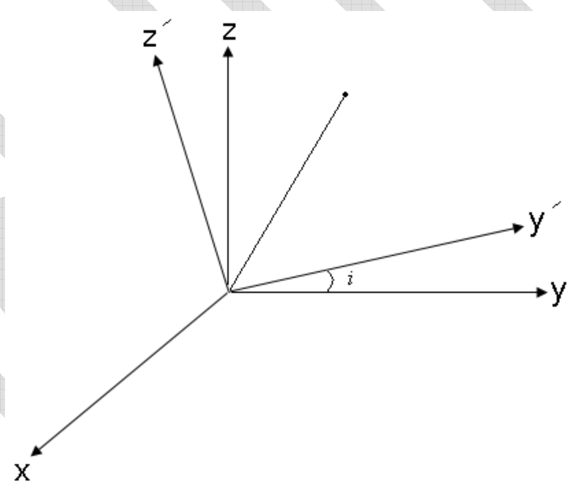
Solution 15



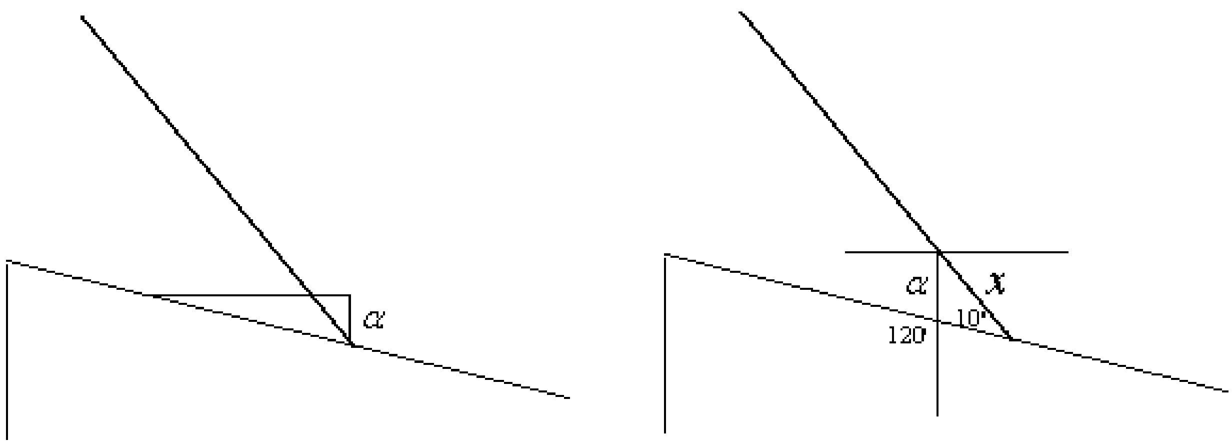
Solution 16



Solution 41



Solution 19



Base Figure